Economics 201A, TA section 7 Anton Cheremukhin

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1. **Contingent commodities**

contingent commodities = goods characterized by type, time, location, state of the world.

Example: Turkey next Thursday at Weyburn if it does not rain. We can think of each such (type/time/location/state) good as a different commodity. This is called the contingent commodity.

2. **The von Neumann-Morgenstern (VNM) utility**

von Neumann-Morgenstern utility describes a utility function (or perhaps a broader class of preference relations) that has the expected utility property: the agent is indifferent between receiving a given bundle or a gamble with the same expected value. It is unique up to an affine transformation. When you deal with risk (states of the world), you cannot apply a positive monotonic transformation, unless it's linear.

VNM representation: $U = \Sigma \pi_i v(c_i)$

Jensen's inequality: $\Sigma \pi_i v(c_i) \leq v(\Sigma \pi_i c_i)$ i.e. $\overline{v(c_i)} \leq v(\overline{c_i})$

(risk-averse preferences) $\Leftrightarrow$ (concave utility function)

No aggregate risk + identical beliefs about probabilities $\Rightarrow \frac{p_i}{p_j} = \frac{\pi_i v'(c_i)}{\pi_j v'(c_j)} = \frac{\pi_i}{\pi_j}$ - full risk sharing.

Aggregate risk, $w_i > w_j \Rightarrow \frac{p_i}{p_j} = \frac{\pi_i v'(c_i)}{\pi_j v'(c_j)} < \frac{\pi_i}{\pi_j}$ - aggregate risk is shared. Picture from the book.

Exercise 1 *Each individual has VNM utility* $v(c) = \ln c$. *Bruin fans believe that UCLA will win with probability* 0.8. *Trojan fans believe that USC will win with probability* 0.6. *Total Bruin fan wealth is* W. *Total Trojan fan wealth is* $3W$. *Let* p_s *be the price of claim to* 1$ *in state* s. *Solve for WE. What if Trojans have wealth* $2W$*?*

State 1: UCLA wins. State 2: USC wins.

Bruins' problem: $U = 0.8 \ln c_1^B + 0.2 \ln c_2^B \to \max$ $\quad s.t.\ p_1 c_1^B + p_2 c_2^B \leq W$

Trojans' problem: $U = 0.4 \ln c_1^T + 0.6 \ln c_2^T \to \max$ $\quad s.t.\ p_1 c_1^T + p_2 c_2^T \leq 3W$

Demands: $c_1^B = 0.8 \frac{W}{p_1}$ $\quad c_1^T = 0.4 \frac{3W}{p_1}$

Total supply in each state of nature: $4W$

WE: $4W = 0.8 \frac{W}{p_1} + 0.4 \frac{3W}{p_1} = 0.2 \frac{W}{p_2} + 0.6 \frac{3W}{p_2} \quad \Rightarrow \quad p_1 = p_2 = 1/2$

William Hill betting agency will set the probabilities equal to 50/50.

$\Rightarrow \quad Win\ odds_i \sim \frac{1}{p_i} - 1$ The win odds will be: 1.00 for both.

Now let Trojans have wealth $2W$. Demands: $c_1^B = 0.8 \frac{W}{p_1}$ $\quad c_1^T = 0.4 \frac{2W}{p_1}$

Total supply in each state of nature: $3W$

WE: $3W = 0.8 \frac{W}{p_1} + 0.4 \frac{2W}{p_1} = 0.2 \frac{W}{p_2} + 0.6 \frac{2W}{p_2}$

$3 = 1.6 \frac{1}{p_1} \quad \frac{1}{p_1} - 1 = 15/8 - 1 = \frac{7}{8} = 0.875$

$3 = 1.4 \frac{1}{p_2} \quad \frac{1}{p_2} - 1 = 15/7 - 1 = \frac{8}{7} = 1.143$

So, when people from UCLA have more money, they win less for one dollar.

(We haven't said anything about the true probabilities so far).

There is no aggregate uncertainty here.

The representative agent logic under full information would lead to prices equal to the true probabilities.

Example 1: Arsenal - 3/10 Draw - 8/1 Newcastle - 7/2

Example 2: Federer - 1/6 Nalbandian - 11/1 Nadal - 10/1 Blake - 10/1 Davydenko - 20/1

3. **Measures of risk-aversion**

Absolute risk aversion: $A(c) = -\frac{v''(c)}{v'(c)}$

CARA: $v(c) = -\frac{1}{A}\exp(-Ac) \quad \Rightarrow \quad -\frac{v''(c)}{v'(c)} = -\frac{(-A)^2\left(-\frac{1}{A}\right)\exp(-Ac)}{-A\left(-\frac{1}{A}\right)\exp(-Ac)} = A$

Relative risk aversion: $R(c) = -c\frac{v''(c)}{v'(c)}$

CRRA: $v(c) = \frac{c^{1-R}}{1-R} \quad \Rightarrow \quad -c\frac{v''(c)}{v'(c)} = -c\frac{(1-R)(-R)\frac{c^{-R-1}}{1-R}}{(1-R)\frac{c^{-R}}{1-R}} = R$

Picture from the book.

4. **Equilibrium with risk neutral consumers and aggregate uncertainty.**

Exercise 2 *There are two states and 1 commodity. The probability of state 1 is 3/4. Alex is risk neutral. Bev is risk averse with VNM utility* $v(c) = \ln(1+c)$. *Aggregate endowment is* $(100, 200)$. *Fully characterize PO allocations. What are the possible state claims price ratios? For what endowments does the risk neutral agent bear all the risk?*

Pareto Optimal allocations:

$\frac{3}{4}c_1^A + \frac{1}{4}c_2^A \to \max \quad s.t. \quad \frac{3}{4}\ln\left(c_1^B + 1\right) + \frac{1}{4}\ln\left(c_2^B + 1\right) \le U$

$s.t. \quad c_1^A + c_1^B \le 100 \quad s.t. \quad c_2^A + c_2^B \le 200$

$\pounds = \frac{3}{4}c_1^A + \frac{1}{4}c_2^A + \mu\left(\frac{3}{4}\ln\left(100 - c_1^A + 1\right) + \frac{1}{4}\ln\left(200 - c_2^A + 1\right)\right) \to \max$

FOC: $\frac{3}{4}\frac{1}{\mu} = \frac{3}{4}\frac{1}{101-c_1^A} \quad \frac{1}{4}\frac{1}{\mu} = \frac{1}{4}\frac{1}{201-c_2^A} \quad \Rightarrow \quad c_2^A = 100 + c_1^A$

This is the 45% line for agent 2 untill it crosses the edge of the Edgeworth box.

By standard reasoning $c_1^A = 0$, $c_2^A \in [0, 100]$ are also PO.

Walrasian Equilibrium:

Alex: $U^A = \frac{3}{4}c_1^A + \frac{1}{4}c_2^A \to \max \quad s.t.\ p_1c_1^A + p_2c_2^A \le p_1w_1^A + p_2w_2^A$

Bev: $U^B = \frac{3}{4}\ln\left(c_1^B + 1\right) + \frac{1}{4}\ln\left(c_2^B + 1\right) \to \max \quad s.t.\ p_1c_1^B + p_2c_2^B \le p_1w_1^B + p_2w_2^B$

$MRS_A = \frac{3/4}{1/4} = 3$ always in the interior. $\quad MRS_B = \frac{3}{4}\frac{1}{101-c_1^A}\frac{4}{1}\frac{201-c_2^A}{1}$

For corner solutions $c_1^A = 0$, $c_2^A \in [0, 100]$, $MRS_B \in \frac{3}{101}\frac{201-[0,100]}{1} = \left[\frac{603}{101}, 3\right]$

This is the range of possible price ratios $\frac{p_1}{p_2}$.

Every time an interior PO is reached, $c_1^B = c_2^B$. So, the risk-free agent bears all the risk.