Economics 201A, TA section 2 Anton Cheremukhin

October 5, 2006

1. **Review of the homework.**

Sketch of Question 1. See Answer Key

2. **Review of the homework.**

Derivation of the CES demands and indirect utility - Question 3.

3. **Review of the homework.**

   Question 5. Derivation of the result.

4. **Homework 1 Problem 4 Fall 2005**

4. Pareto Efficient Allocations

Consumer h, $h = A, B$ has a utility function $U^h(x_1^h, x_2^h) = (a_1^h + x_1^h)^{\alpha_1}(a_2^h + x_1^h)^{\alpha_2}$.

The aggregate endowment vector is ω.

(a) The allocation $\bar{x}^A, \bar{x}^B$ is Pareto Efficient if $(\bar{x}^A, \bar{x}^B)$ is the solution to the following problem.

$$\underset{x^A, x^B}{Max}\{U^A(x^A) \mid x^A + x^B \le \omega,\ U^B(x^B) \ge U^B(\bar{x}^B)\}$$

Explain why this must be true by drawing an Edgeworth Box Diagram..

(b) Suppose $a^A = a^B = 0$. Show that the Pareto Efficient allocations in the interior of the Edgeworth Box lie on the diagonal.

(c) For all vectors $a^A, a^B \ge 0$ show that the PE allocations in the interior of the box lie on a line.

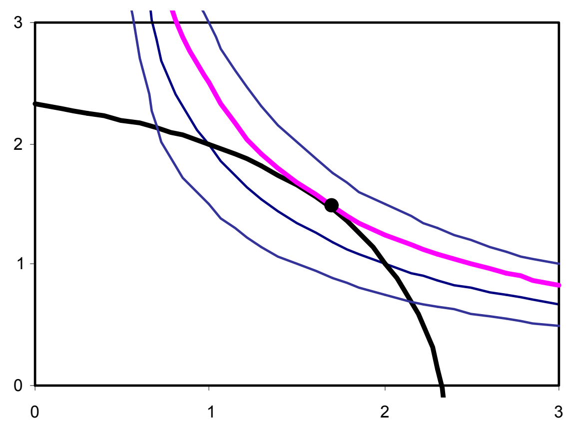
(d) Define $y^h = a^h + x^h$, $h = A, B$. Then $U^h = (y_1^h)^{\alpha_1}(y_2^h)^{\alpha_2}$.

Note that $y^A + y^B \le a^A + a^B + \omega$. Then draw a big Edgeworth Box and depict the PE allocations in this diagram. Depict a smaller Edgeworth Box showing the feasible y allocations inside the big one if $a^A = a^B = (2,1)$

(e) Are there any values of a^A, a^B for which there are no PE allocations in the interior of the Edgeworth Box? Explain.

(a) Here is the definition of PE allocation:
$\max\{ \quad \alpha_1 \ln(a_1+x_1)+\alpha_2\ln(a_2+x_2) \quad | \quad x_1+y_1 \le w_1, \;\; x_2+y_2\le w_2,$
$\alpha_1\ln(b_1+y_1)+\alpha_2\ln(b_2+y_2)\ge U_2, \quad x_i \ge 0, \quad y_i \ge 0 \quad \}$

It means that given some indifference curve of the second guy we search for a point on it, that gives the other guy maximum utility. At that point the indifference curves should be tangent if it is in the interior of the Edgeworth box.



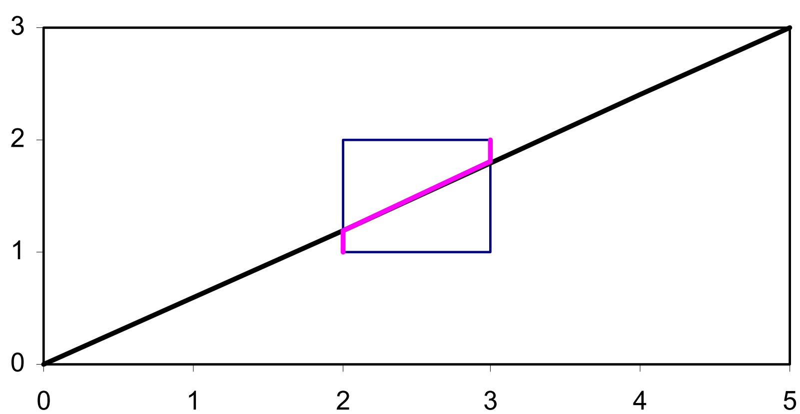
The definition simplifies to for the interior cases:
$\max\{\alpha_1\ln(a_1+w_1-y_1)+\alpha_2\ln(a_2+w_2-y_2)| \;\; \alpha_1\ln(b_1+y_1)+\alpha_2\ln(b_2+y_2)\ge U_2, y_i \ge 0\}$
Lagrangian:$\pounds = \alpha_1\ln(a_1+w_1-y_1)+\alpha_2\ln(a_2+w_2-y_2)+\lambda(\alpha_1\ln(b_1+y_1)+\alpha_2\ln(b_2+y_2)-U_2)$
FOC for interior: $\quad \lambda = \frac{y_1+b_1}{a_1+w_1-y_1} = \frac{y_2+b_2}{a_2+w_2-y_2} \;\; \Rightarrow$
$y_1(a_2+w_2+b_1)+b_1(a_2+w_2) = y_2(a_1+w_1+b_2)+b_2(a_1+w_1)$

(b) For the case $a_1 = b_1 = 0$ we get: $y_1 w_2 = y_2 w_1$
This is the diagonal of the Edgeworth box.

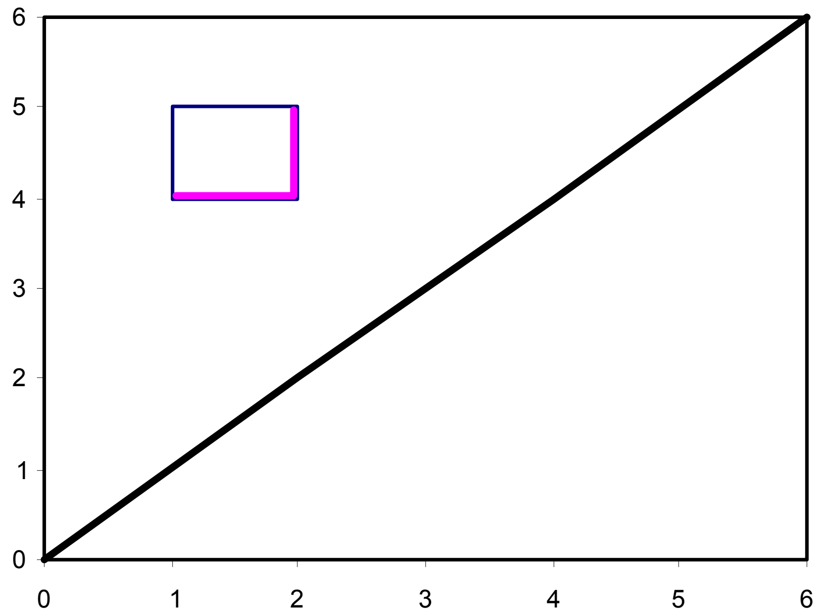
(c) The condition derived from FOC is a line.

(d) Redefining $z_i = b_i + y_i$ gives: $\frac{z_1}{a_1+w_1+b_1-z_1} = \frac{z_2}{a_2+w_2+b_2-z_2} \Rightarrow$
$$z_1(a_2+w_2+b_2) = z_2(a_1+w_1+b_1)$$

That means our Edgeworth box is a subset of a bigger Edgeworth box in z_i coordinates in which PE allocations are on the diagonal. For $a=(2;1) \quad b=(2;1) \quad w=(1;1)$ the picture looks like this:



(e) The example values are: $a = (1;4) \quad b = (4;1) \quad w = (1;1)$.



5. **List of facts from Chapter 4.**

Primal problem $\max_x \{u(x) | p \cdot x \leq I\}$
$u(x)$ - direct utility
$x(p, I)$ - (marshallian) demand functions
$v(p, I) = u(x(p, I)) = \max_x \{u(x) | p \cdot x \leq I\}$ - indirect utility

- Continuous
- Homogeneous degree zero in (p,I)
- Strictly increasing in I
- Decreasing in p
- quasiconvex in (p,I)

Roy's identity: $x_i(p, I) = -\frac{\partial v(p,I)/\partial p_i}{\partial v(p,I)/\partial I}$
Dual problem $\min_x \{p \cdot x | u(x) \geq U\}$
$x^c(p, U)$ - (hicksian) compensated demand functions
$e(p, U) = p \cdot x^c(p, U) = \min_x \{p \cdot x | u(x) \geq U\}$ - expenditure function

- continuous
- strictly increasing in U
- increasing in p

- homogeneous of degree 1 in p
- concave in p

Sheppard's lemma: $\partial e(p,U)/\partial p_i = x_i^c(p,U)$
Relationships:
$e(p,v(p,I)) = I \qquad v(p,e(p,U)) = U$
$x_i(p,I) = x_i^c(p,v(p,I)) \qquad x_i^c(p,U) = x_i(p,e(p,U))$
Slutsky equation: $\frac{\partial x_i(p,I)}{\partial p_j} = \left.\frac{\partial x_i^c(p,U)}{\partial p_j}\right|_U - x_j(p,I)\frac{\partial x_i(p,I)}{\partial I} = SE + IE$
Elasticities:
$E[x,I] = \frac{I}{x}\frac{\partial x}{\partial I} \qquad$ income share: $k_i = \frac{p_i x_i}{I}$
Engel aggregation (Lemma, 4-2 page 4): $\sum_{i=1}^{n} k_i E[x_i, I] = 1$
"weighted income elasticities of demand sum up to 1"
Cournot aggregation (Lemma, 4-2 page 10): $\sum_{i=1}^{n} k_i E[x_i, p_j] + k_j = 0$ for all j.
For the compensated demand if follows that $\sum_{i=1}^{n} k_i E[x_i^c, p_j] = 0$
"weighted price elasticities of compensated demand sum up to zero"
$E[x_i, p_j]$ - cross-price elasticities, $E[x_i, p_i]$ - own-price elasticity
Slutsky equation: $E[x_i, p_i] = E[x_i^c, p_i] - k_1 E[x_i, I]$
Elasticity of substitution (2 goods): $E\left[\frac{x_2^c}{x_1^c}, p_1\right] = p_1 \frac{\partial}{\partial p_1} \ln \frac{x_2^c}{x_1^c}$
$\sigma \equiv E\left[\frac{x_2^c}{x_1^c}, p_1\right] = \frac{E\left[x_2^c, p_1\right]}{k_1} = E[x_2^c, p_1] - E[x_1^c, p_1] = E\left[\frac{x_1^c}{x_2^c}, p_2\right]$
Using this, one can show, that $E[x_1^c, p_1] = -k_2\sigma$.
Therefore, $E[x_1, p_1] = -(1-k_1)\sigma - k_1 E[x_i, I]$ - decomposition of own price elasticity.
CES: $\left(x_1^{1-\frac{1}{\sigma}} + x_2^{1-\frac{1}{\sigma}}\right)^{\frac{1}{1-\frac{1}{\sigma}}}$ - has constant elasticity of substitution.

6.* **Exercise**

4. Elasticity of consumption ratios

A consumer with CES preferences faces a price vector p and has income I.

(a) For the CES family solve for the optimal ratio of demand $\frac{x_j}{x_i}$.

(b) How does this vary with income?

(c) What is the elasticity of this ratio with respect to a change in the price of commodity i and commodity j?