Economics 201A, TA section 1 Anton Cheremukhin

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1. **Primitive notions**

Consumption set: $\emptyset \neq X \subseteq R_+^n, closed, convex, 0 \in X$
Binary relations of the type: $x^1 \succeq x^2$ (is at least as good as) - represent preference relations.
Strict relations: $x^1 \succ x^2 \Leftrightarrow [x^1 \succeq x^2$ and $x^2 \not\succeq x^1]$ (is better than)
Indifference: $x^1 \sim x^2 \Leftrightarrow [x^1 \succeq x^2$ and $x^2 \succeq x^1]$ (is indifferent between)
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Axioms of consumer choice (preferences relations are):
A1: completeness: $\forall x^1, \forall x^2 : x^1 \succeq x^2$ or $x^2 \succeq x^1$
A2: transitivity: $x^1 \succeq x^2$ and $x^2 \succeq x^3 \Rightarrow x^1 \succeq x^3$
A3: continuity: $\forall x$ sets $(\succeq x)$ and $(\preceq x)$ are closed in R_+^n
A4': local non-satiation: $\forall x^0 \in R_+^n, \forall \varepsilon > 0 \exists x \in B_\varepsilon(x^0) \subset R_+^n : x > x^0$
A4: strict monotonicity: $\forall x^0, x^1 \in R_+^n : \begin{cases} \text{if } x^0 \geq x^1 \Rightarrow x^0 \succeq x^1 \\ \text{if } x^0 >> x^1 \Rightarrow x^0 \succ x^1 \end{cases}$
A5': convexity: if $x^1 \succeq x^0 \Rightarrow \forall t \in [0,1] : tx^1 + (1-t)x^0 \succeq x^0$
A5: strict convexity: if $x^1 \succeq x^0, x^1 \neq x^0 \Rightarrow \forall t \in (0,1) : tx^1 + (1-t)x^0 \succ x^0$
Observation: A4 $\Rightarrow$ A4', A5 $\Rightarrow$ A5'

2. **Utility functions**

Real-valued function $U : R_+^n \longrightarrow R$ represents the preference relations $(\succeq)$
if $\forall x^0, x^1 \in R_+^n : u(x^0) \geq u(x^1) \Leftrightarrow x^0 \succeq x^1$
Theorem: A1,A2,A3,A4 guarantee $\exists U$ - continuous real-valued utility function, representing $(\succeq)$
It is not unique - it is said to be unique up to a positive monotonic transformation.
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Interpretation: preferences are (strictly) convex $\Leftrightarrow$ the utility function is (strictly) quasi-concave
preferences are strictly monotonic $\Leftrightarrow$ the utility function is strictly increasing
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Quasi-concavity: $A = \{x : u(x) \geq t\}$ (upper countersets) are convex for any t.
Equivalent to: $\forall x, x' \in R_+^n, \forall t, \forall \lambda \in [0,1] : \{u(x) \geq t$ and $u(x') \geq t\} \Rightarrow u(\lambda x + (1-\lambda)x') \geq t$

3. **CES function**

$U(x_1, x_2) = \left(x_1^{1-\frac{1}{\sigma}} + x_2^{1-\frac{1}{\sigma}}\right)^{\frac{1}{1-\frac{1}{\sigma}}} \qquad x_2 = \left(U^{1-\frac{1}{\sigma}} - x_1^{1-\frac{1}{\sigma}}\right)^{\frac{1}{1-\frac{1}{\sigma}}}$
Case 1: $0 < \sigma < 1 :$ The IC has limiting values: $U = a$, $x_1 \to \infty \Rightarrow x_1 \to a$.
Case 2: $\sigma > 1 :$ The IC hits the axis: $U = a$, $x_1 \to \infty \Rightarrow x_1 \to -\infty$.
Case 3: $\sigma \to 0 :$ The IC is Leontieff Case 4: $\sigma \to 1 :$ The IC is Cobb-Douglas
Case 5: $\sigma \to \infty :$ The IC is linear Case 6: $\sigma < 0 :$ The IC is convex

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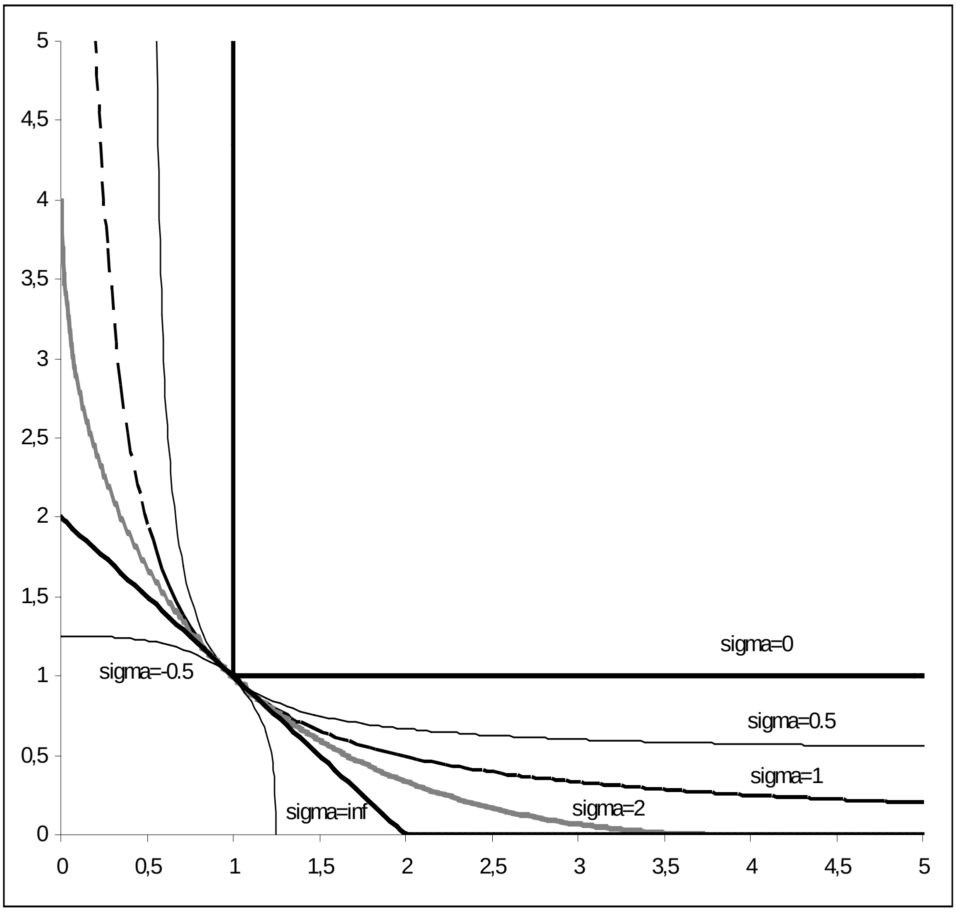

sigma=0
sigma=-0.5
sigma=0.5
sigma=1
sigma=2
sigma=inf