STRUCTURAL TRANSFORMATION AND DEVELOPMENT: DEMAND OR SUPPLY STORY?

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Abstract. What is driving structural transformation? Some argue that it can be driven by different income elasticities, and others say that differences in productivity growth are the source of labor reallocation. We present a unifying framework which allows us to quantify the importance of supply and demand mechanisms for structural transformation and see how these forces change over time.

PRELIMINARY AND INCOMPLETE

I. Introduction

There are two well-known facts about structural transformation. During the first stage of development labor moves out of the agricultural sector into manufacturing and services. Later in the development process labor moves out of both agriculture and manufacturing into services. What is driving this transformation? Some argue that it can be driven by different income elasticities, and others say that differences in productivity growth are the source of labor reallocation. We present a unifying framework which allows us to quantify the importance of supply and demand mechanisms for structural transformation and see how these forces change over time.

We find that the behavior of both preferences and technology in the service sector is non-monotone. However, the change in preferences, not technology, is the main cause of the reallocation of labor into the service sector.

The existing literature assumes that together with structural transformation there is a balanced growth path. In order to achieve both they impose strong theoretical assumptions (see Kongsamut, Rebelo and Xie (2001, [2]) and Ngai and Pissarides (2007, [3])), which narrow the capacity of the model to capture the behavior of the data.

Our methodology allows us to avoid making strong theoretical assumptions to guarantee the existence of a balanced growth path. Despite the absence of balanced growth, our model satisfies the Kaldor facts. Under perfect foresight we use the first-order conditions and resource constraints together with data on labor shares, output and capital to recover the preference parameters and sectorial technologies. We then run counterfactual experiments to evaluate the importance of demand and supply mechanisms behind structural transformation.

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There are two related papers. Herrendorf, Rogerson, and Valentinyi (2008) estimate preference and technology parameters using postwar U.S. data and find that neither story is enough on its own. Buera and Kaboski (2008, [1]) integrate demand and supply side explanations and find that the two stories together are not enough when preference parameters are constant. Our model allows us to identify the changes in preference and technology parameters and evaluate the importance of those changes.

The paper is organized as follows. Section 2 describes the model, section 3 explains the methodology, section 4 shows our results and section 5 concludes.

II. Model

We use a standard three-sector model with Stone-Geary preferences and Cobb-Douglas production functions. We allow the Stone-Geary subsistence levels of consumption as well as productivity levels to be time-varying. The three sectors represent agriculture, manufacturing and services. We assume that investment goods are produced in the manufacturing sector. To be able to study the behavior of relative prices we present a decentralized version of the model. There is a representative household that maximizes the expected discounted present value of the utility function:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} N_t \frac{\left(\left(c_t^A - \gamma_t^A\right)^{\eta_A} \left(c_t^M\right)^{\eta_M} \left(c_t^S - \gamma_t^S\right)^{\eta_S}\right)^{1-\theta} - 1}{1-\theta} \rightarrow \max_{c_t^i, K_{t+1}} \tag{1}$$

subject to the budget constraint

$$p_t^A c_t^A N_t + c_t^M N_t + p_t^S c_t^S N_t + \left(K_{t+1} - (1-\delta) K_t\right) = r_t K_t + w_t L_t, \tag{2}$$

where N_t is population, c_t^i is the per capita consumption of good i, γ_t^i is the subsistence level of consumption of good i, p_t^i is the relative price of good i with respect to manufacturing, K_t is the capital stock, r_t is the return to capital, w_t is the wage rate and L_t is the labor force. Finally, $i = A, M, S$ where A stands for agriculture, M stands for manufacturing and S stands for services.

Production in each sector is done by a competitive firm which maximizes period-by-period profits:

$$\pi_t^i = p_t^i Y_t^i - r_t K_t^i - w_t L_t^i \tag{3}$$

subject to

$$Y_t^i = A_t^i \left(K_t^i\right)^{\alpha_i} \left(L_t^i\right)^{1-\alpha_i}, \tag{4}$$

where Y_t^i is output in sector i, A_t^i is productivity in sector i, K_t^i is capital used in sector i and L_t^i is labor used in sector i. The market clearing conditions are given by:

$$K_t^A + K_t^M + K_t^S = K_t, \tag{5}$$

$$L_t^A + L_t^M + L_t^S = L_t, \tag{6}$$

$$c_t^A N_t = Y_t^A, \tag{7}$$

$$c_t^M N_t + K_{t+1} - (1-\delta) K_t = Y_t^M, \tag{8}$$

$$c_t^S N_t = Y_t^S. \tag{9}$$

III. Methodology

As long as we have non-zero subsistence levels of consumption and they are not directly linked to productivities, the model does not have a balanced growth path (see Buera, Kaboski (2008, [1])). Therefore we cannot solve for the policy functions of the model. To get around this problem, we assume perfect foresight. We use the first-order conditions and resource constraints together with data on labor shares, output and capital to recover the preference parameters and sectorial technologies.

Notice, that capital and labor are mobile across sectors, hence, the factor prices are equalized. Therefore, capital shares depend on capital intensities and labor shares in the three sectors:

$$K_t^i = \frac{\frac{\alpha_i}{1-\alpha_i} L_t^i}{\frac{\alpha_A}{1-\alpha_A} L_t^A + \frac{\alpha_M}{1-\alpha_M} L_t^M + \frac{\alpha_S}{1-\alpha_S} L_t^S} K_t. \tag{10}$$

We use data on labor shares and aggregate capital stock to compute the levels of capital used in the three sectors. We combine it with data on real output to compute the levels of technology:

$$A_t^i = \frac{Y_t^i}{(K_t^i)^{\alpha_i} (L_t^i)^{1-\alpha_i}} \tag{11}$$

We use the first order conditions for consumption to pin down the subsistence levels in agriculture and services (see appendix):

$$\gamma_t^i = \frac{Y_t^i}{N_t} \left[1 - \frac{\eta_i}{\eta_M} \frac{1-\alpha_i}{1-\alpha_M} \frac{L_t^M}{L_t^i} \frac{Y_t^M - K_{t+1} + (1-\delta) K_t}{Y_t^M} \right] \tag{12}$$

We construct data for output, labor and capital stock for ten-year periods from 1869 until 2008. Labor shares are measured using the number of employees, output is the real GDP index, and the real capital stock is constructed from the real gross private domestic investment. We use Kendrick (1961) to construct data for 1869-1957 and BEA and BLS for 1929-2008.

We calibrate parameters to the values shown in Table 1. The discount factor and the depreciation rate correspond to a yearly rate of 4% and 6% respectively. We assume that preferences are logarithmic. We take the values of capital intensities from Kendrick (1961).

β	δ	α_A	α_M	α_S	η_A	η_M	η_S	θ
0.665	0.46	0.36	0.23	0.15	0.05	0.32	0.63	1

Table 1. Calibrated parameters

IV. Results

The top panel of Figure 1 shows the data for real output and labor shares. One can observe that the labor share in agriculture is constantly decreasing, the labor share in manufacturing is increasing in the first half of the sample and decreasing thereafter, the labor share in services is constantly increasing.

The bottom panel shows the results for the productivities and preference parameters as well as consumption. We see that productivity in the services sector slows

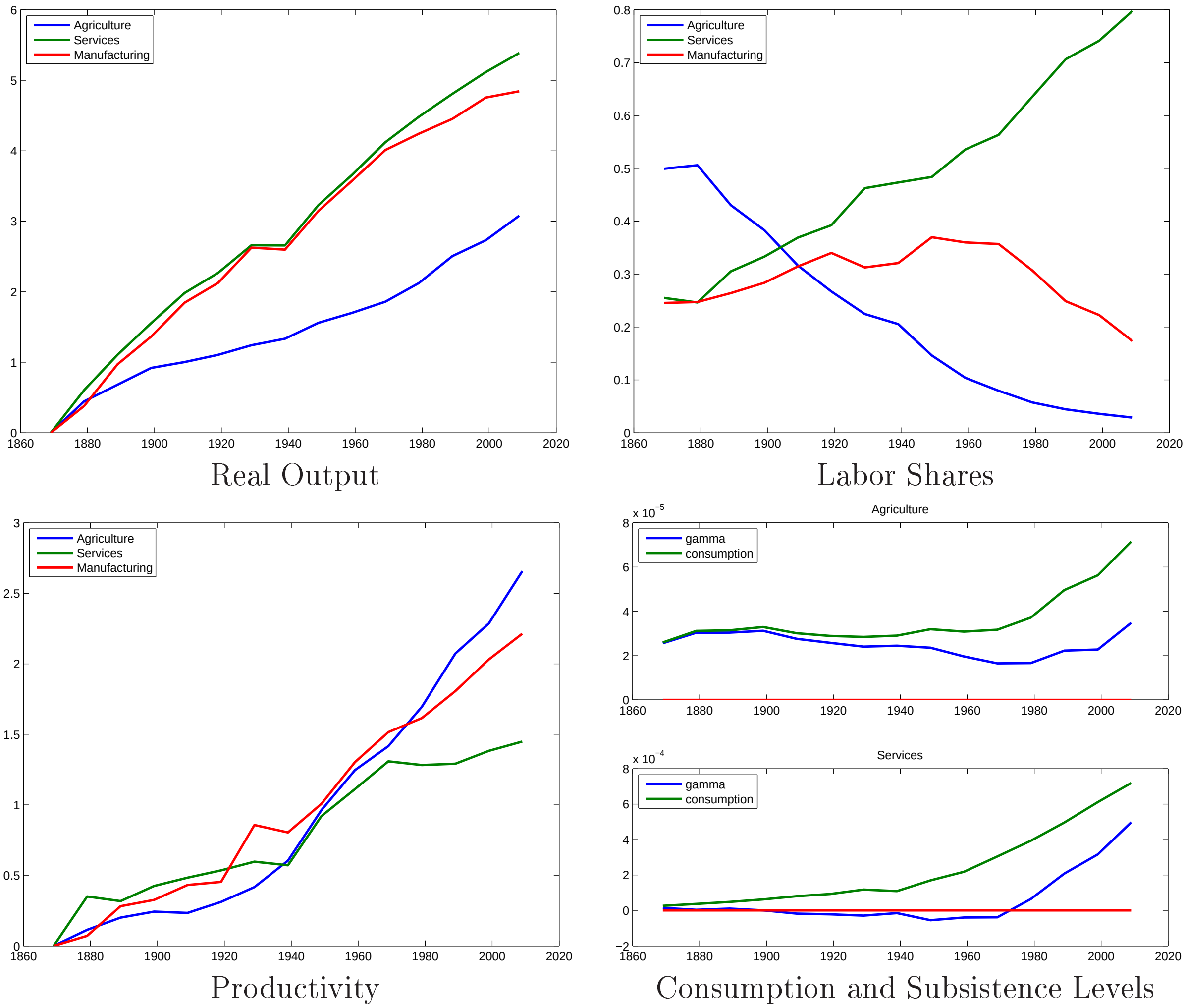

FIGURE 1. Data, Consumption, Productivity and Preferences

down around 1970 and at the same time the subsistence level of consumption of services increases significantly. Each of these facts could potentially explain the observed changes in the labor shares.

In order to quantify the importance of each mechanism we conduct two counterfactual experiments. In the first one we assume that the subsistence level remains at its year 1970 level, it does not increase. In the second counterfactual experiment we assume that productivity in services continues to grow at the same rate after 1970 as in the previous decade. We compute the alternative paths of all endogenous variables in the model, assuming the path of capital does not change.

Figure 2 shows the paths of output, consumption and labor shares when the subsistence level of consumption of services remains at its 1970 level. Notice that if this were the case then there would be no reallocation of labor from manufacturing to services. Furthermore, consumption and output of services would decrease and consumption and output of the other two sectors would increase. Hence, the shift in preferences alone is able to explain all the labor reallocation.

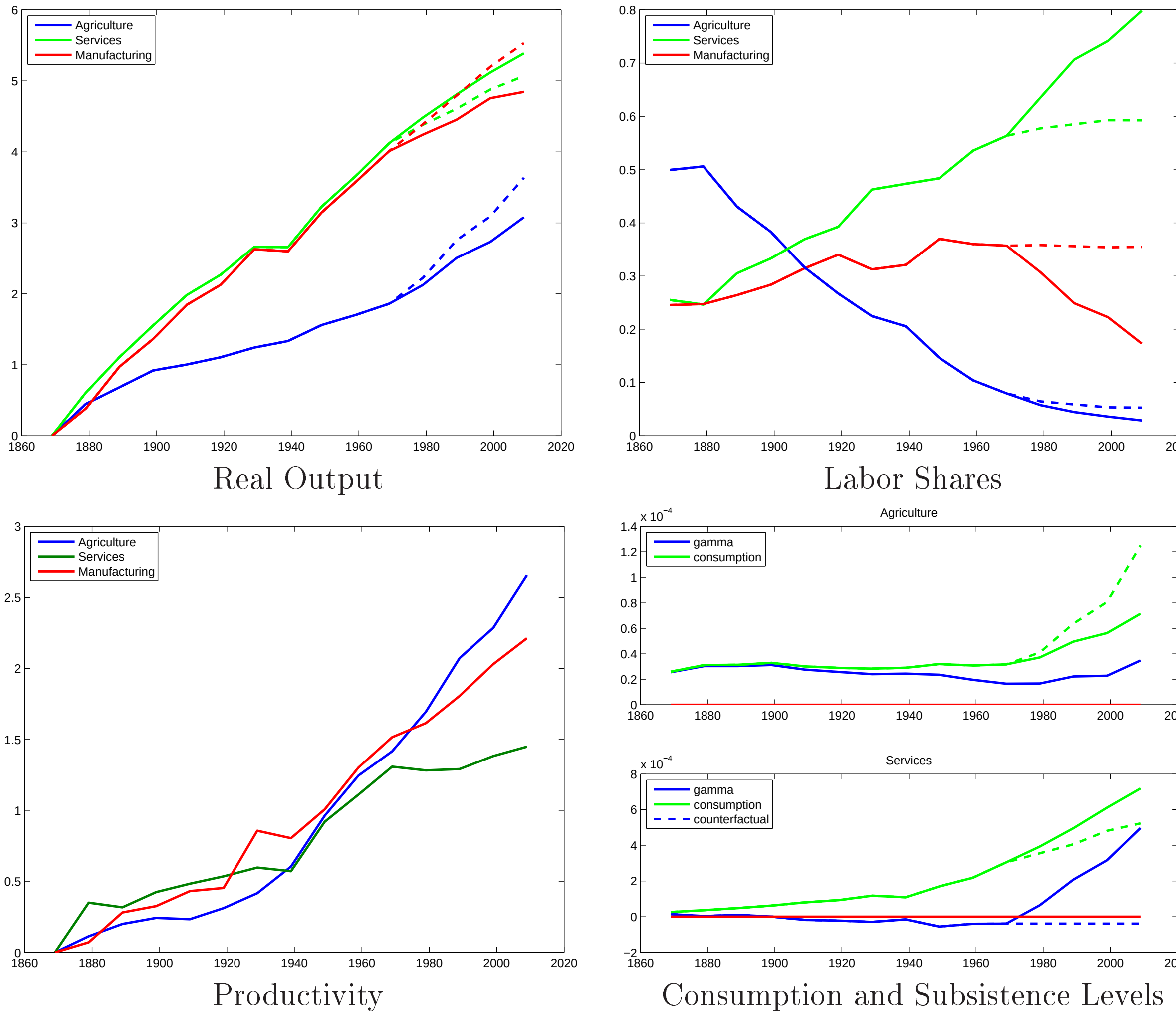

FIGURE 2. Counterfactual Subsistence Level of the Service Sector

Figure 3 shows the paths of output, consumption and labor shares when there is no slowdown in productivity growth of services after 1970. The alternative path for productivity can explain about one third of the observed reallocation of labor from manufacturing to services. It also generates an increase in the growth rates of consumption and output in the three sectors. Hence, the slowdown in the productivity growth of the service sector, by itself, cannot explain the observed labor reallocation.

V. CONCLUSION

Structural transformation is characterized by labor reallocation first from the agricultural sector into manufacturing and services and then out of both agriculture and manufacturing into services. Existing literature says that the transformation can be driven by differences in income elasticities or by differences in productivity growth in the three sectors. We present a unifying framework which allows us to quantify the importance of supply and demand mechanisms for structural transformation and see how these forces change over time.

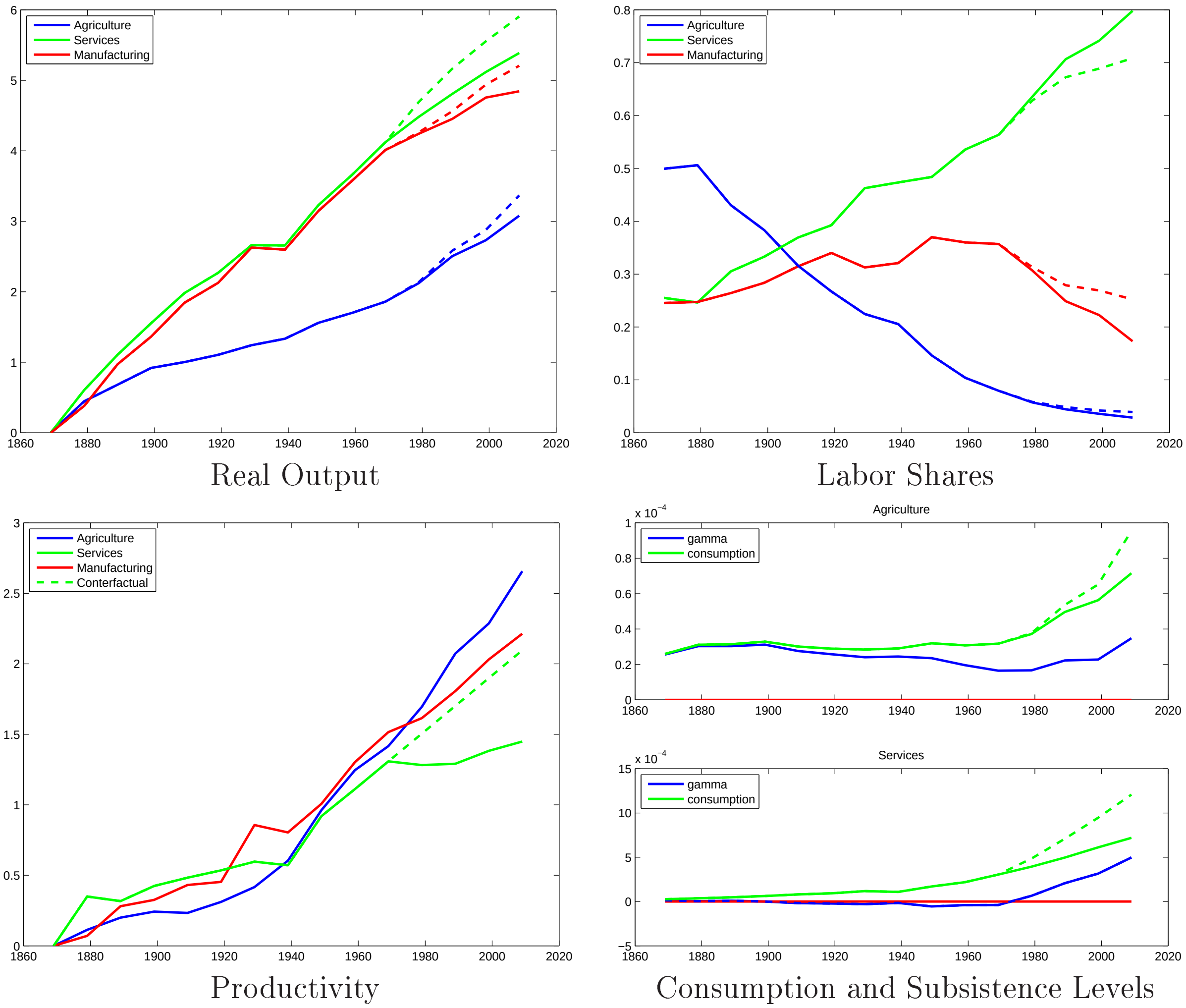

FIGURE 3. Counterfactual Productivity of the Service Sector

We find that driving forces change over time. TFP in the service sector slows down around 1970 and the need for services increases drastically at the same time. We find that preferences have a clear effect on labor shares while technology has only a small effect on labor shares, mainly affecting output.

Directions for future work: a more general structure of preferences and technology, study implications for relative prices, importance of health services and women moving into the labor force in explaining the kink in preferences. We also plan to refine data sources.

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VI. Appendix

VI.1. First-Order Conditions.

First order conditions for the household:

$$\sum_{s=t}^{\infty}\left[\beta^{s-t}N_s u\left(c_s\right)-\beta^{s-t}\lambda_s\left(p_s^A c_s^A N_s+c_s^M N_s+p_s^S c_s^S N_s+\left(K_{s+1}-\left(1-\delta\right)K_s\right)-r_s K_s-w_s L_s\right)\right]\rightarrow\max_{c_s^i,K_{s+1}} \tag{13}$$

$$\beta^{s-t}N_s\frac{\partial u\left(c_s\right)}{\partial c_s}\frac{\partial c_s}{\partial c_s^A}-\beta^{s-t}\lambda_s p_s^A N_s=0 \tag{14}$$

$$\beta^{s-t}N_s\frac{\partial u\left(c_s\right)}{\partial c_s}\frac{\partial c_s}{\partial c_s^M}-\beta^{s-t}\lambda_s N_s=0 \tag{15}$$

$$\beta^{s-t}N_s\frac{\partial u\left(c_s\right)}{\partial c_s}\frac{\partial c_s}{\partial c_s^S}-\beta^{s-t}\lambda_s p_s^S N_s=0 \tag{16}$$

$$-\beta^{s-t}\lambda_s+\beta^{s+1-t}E_s\lambda_{s+1}\left(1-\delta+r_{s+1}\right)=0 \tag{17}$$

Hence,

$$\lambda_t p_t^A=\frac{\partial u\left(c_t\right)}{\partial c_t}\frac{\partial c_t}{\partial c_t^A}=c_t^{1-\theta}\frac{\eta_A}{c_t^A-\gamma_t^A} \tag{18}$$

$$\lambda_t=\frac{\partial u\left(c_t\right)}{\partial c_t}\frac{\partial c_t}{\partial c_t^M}=c_t^{1-\theta}\frac{\eta_M}{c_t^M} \tag{19}$$

$$\lambda_t p_t^S=\frac{\partial u\left(c_t\right)}{\partial c_t}\frac{\partial c_t}{\partial c_t^S}=c_t^{1-\theta}\frac{\eta_S}{c_t^S-\gamma_t^S} \tag{20}$$

$$\lambda_t=E_t\lambda_{t+1}\beta\left(1-\delta+r_{t+1}\right) \tag{21}$$

Therefore,

$$\frac{\eta_A}{\eta_M}\frac{c_t^M}{c_t^A-\gamma_t^A}=p_t^A \tag{22}$$

$$\frac{\eta_S}{\eta_M}\frac{c_t^M}{c_t^S-\gamma_t^S}=p_t^S \tag{23}$$

$$1=E_t\left[\frac{c_{t+1}^{1-\theta}}{c_t^{1-\theta}}\frac{c_t^M}{c_{t+1}^M}\beta\left(1-\delta+r_{t+1}\right)\right] \tag{24}$$

First order conditions for the firms:

$$r_t=p_t^A\alpha_A\frac{Y_t^A}{K_t^A}=\alpha_M\frac{Y_t^M}{K_t^M}=p_t^S\alpha_S\frac{Y_t^S}{K_t^S} \tag{25}$$

$$(13-15) \tag{26}$$

$$w_t=p_t^A\left(1-\alpha_A\right)\frac{Y_t^A}{L_t^A}=\left(1-\alpha_M\right)\frac{Y_t^M}{L_t^M}=p_t^S\left(1-\alpha_S\right)\frac{Y_t^S}{L_t^S} \tag{27}$$

VI.2. **Summary.**

$$(1) \qquad c_t: \qquad c_t = \left(c_t^A - \gamma_t^A\right)^{\eta_A} \left(c_t^M\right)^{\eta_M} \left(c_t^S - \gamma_t^S\right)^{\eta_S} \tag{28}$$

$$(2) \qquad Y_t^A: \qquad Y_t^A = A_t^A \left(K_t^A\right)^{\alpha_A} \left(L_t^A\right)^{1-\alpha_A} \tag{29}$$

$$(3) \qquad Y_t^M: \qquad Y_t^M = A_t^M \left(K_t^M\right)^{\alpha_M} \left(L_t^M\right)^{1-\alpha_M} \tag{30}$$

$$(4) \qquad Y_t^S: \qquad Y_t^S = A_t^S \left(K_t^S\right)^{\alpha_S} \left(L_t^S\right)^{1-\alpha_S} \tag{31}$$

$$(5) \qquad r_t: \qquad K_t^A + K_t^M + K_t^S = K_t \tag{32}$$

$$(6) \qquad w_t: \qquad L_t^A + L_t^M + L_t^S = L_t \tag{33}$$

$$(7) \qquad c_t^A: \qquad c_t^A N_t = Y_t^A \tag{34}$$

$$(8) \qquad c_t^M: \qquad c_t^M N_t + K_{t+1} - (1-\delta) K_t = Y_t^M \tag{35}$$

$$(9) \qquad c_t^S: \qquad c_t^S N_t = Y_t^S \tag{36}$$

$$(10) \qquad p_t^A: \qquad \frac{\eta_A}{\eta_M} \frac{c_t^M}{c_t^A - \gamma_t^A} = p_t^A \tag{37}$$

$$(11) \qquad p_t^S: \qquad \frac{\eta_S}{\eta_M} \frac{c_t^M}{c_t^S - \gamma_t^S} = p_t^S \tag{38}$$

$$(12) \qquad K_{t+1}: \qquad 1 = E_t \left[\frac{c_{t+1}^{1-\theta}}{c_t^{1-\theta}} \frac{c_t^M}{c_{t+1}^M} \beta \left(1 - \delta + r_{t+1}\right)\right] \tag{39}$$

$$(13) \qquad K_t^A: \qquad r_t = p_t^A \alpha_A \frac{Y_t^A}{K_t^A} \tag{40}$$

$$(14) \qquad K_t^M: \qquad r_t = \alpha_M \frac{Y_t^M}{K_t^M} \tag{41}$$

$$(15) \qquad K_t^S: \qquad r_t = p_t^S \alpha_S \frac{Y_t^S}{K_t^S} \tag{42}$$

$$(16) \qquad L_t^A: \qquad w_t = p_t^A \left(1 - \alpha_A\right) \frac{Y_t^A}{L_t^A} \tag{43}$$

$$(17) \qquad L_t^M: \qquad w_t = \left(1 - \alpha_M\right) \frac{Y_t^M}{L_t^M} \tag{44}$$

$$\tag{45}$$

$$(18) \qquad L_t^S: \qquad w_t = p_t^S \left(1 - \alpha_S\right) \frac{Y_t^S}{L_t^S} \tag{46}$$

18 prices and allocations: $\left\{K_t^{A,M,S}, L_t^{A,M,S}, Y_t^{A,M,S}, c_t^{A,M,S}, K_{t+1}, c_t, p_t^{A,S}, r_t, w_t\right\}$ with $\left\{A_t^{A,M,S}, \gamma_t^{A,S}, N_t, L_t\right\}$ exogenously given

VI.3. **Derivation of the Solution Algorithm.** We know $Y_t^A, Y_t^M, Y_t^S, L_t^A, L_t^M, L_t^S, L_t, N_t$. Can we solve for the wedges of interest $\left\{A_t^{A,M,S}, \gamma_t^{A,S}\right\}$?

Assume:$\eta_A + \eta_M + \eta_S = 1$. First of all: $L_t = L_t^A + L_t^M + L_t^S$

Substitute out prices r_t, w_t, p_t^A and p_t^S

$$\frac{1-\alpha_M}{1-\alpha_A}\frac{L_t^A}{L_t^M}\frac{Y_t^M}{Y_t^A} = p_t^A = \frac{\alpha_M}{\alpha_A}\frac{K_t^A}{K_t^M}\frac{Y_t^M}{Y_t^A} \tag{47}$$

$$\frac{1-\alpha_M}{1-\alpha_S}\frac{L_t^S}{L_t^M}\frac{Y_t^M}{Y_t^S} = p_t^S = \frac{\alpha_M}{\alpha_S}\frac{K_t^S}{K_t^M}\frac{Y_t^M}{Y_t^S} \tag{48}$$

Substitute out consumptions:

$$c_t^A = \frac{Y_t^A}{N_t} \tag{49}$$

$$c_t^M = \frac{Y_t^M - K_{t+1} + (1-\delta)K_t}{N_t} \tag{50}$$

$$c_t^S = \frac{Y_t^S}{N_t} \tag{51}$$

$$c_t = \left(c_t^A - \gamma_t^A\right)^{\eta_A}\left(c_t^M\right)^{\eta_M}\left(c_t^S - \gamma_t^S\right)^{\eta_S} = \frac{\left(Y_t^A - N_t\gamma_t^A\right)^{\eta_A}\left(Y_t^M - K_{t+1} + (1-\delta)K_t\right)^{\eta_M}\left(Y_t^S - N_t\gamma_t^S\right)^{\eta_S}}{N_t} \tag{52}$$

Thus,

$$\frac{\eta_A}{\eta_M}\frac{Y_t^M - K_{t+1} + (1-\delta)K_t}{Y_t^A - N_t\gamma_t^A} = p_t^A = \frac{1-\alpha_M}{1-\alpha_A}\frac{L_t^A}{L_t^M}\frac{Y_t^M}{Y_t^A} \tag{53}$$

$$\frac{\eta_S}{\eta_M}\frac{Y_t^M - K_{t+1} + (1-\delta)K_t}{Y_t^S - N_t\gamma_t^S} = p_t^S = \frac{1-\alpha_M}{1-\alpha_S}\frac{L_t^S}{L_t^M}\frac{Y_t^M}{Y_t^S} \tag{54}$$

$$1 = E_t\left[\begin{array}{c}\left(\frac{\left(Y_{t+1}^A - N_{t+1}\gamma_{t+1}^A\right)^{\eta_A}\left(Y_{t+1}^M - K_{t+2} + (1-\delta)K_{t+1}\right)^{\eta_M}\left(Y_{t+1}^S - N_{t+1}\gamma_{t+1}^S\right)^{\eta_S}}{\left(Y_t^A - N_t\gamma_t^A\right)^{\eta_A}\left(Y_t^M - K_{t+1} + (1-\delta)K_t\right)^{\eta_M}\left(Y_t^S - N_t\gamma_t^S\right)^{\eta_S}}\frac{N_t}{N_{t+1}}\right)^{1-\theta} \\ * \frac{\frac{Y_t^M - K_{t+1} + (1-\delta)K_t}{N_t}}{\frac{Y_{t+1}^M - K_{t+2} + (1-\delta)K_{t+1}}{N_{t+1}}}\beta\left(1 - \delta + \alpha_M\frac{Y_{t+1}^M}{K_{t+1}^M}\right)\end{array}\right] \tag{55}$$

Then, we use the fact that from equations above it follows, that:

$$K_t^A = \frac{\alpha_A}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_A}\frac{L_t^A}{L_t^M}K_t^M \tag{56}$$

$$K_t^S = \frac{\alpha_S}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_S}\frac{L_t^S}{L_t^M}K_t^M \tag{57}$$

to substitute:

$$K_t = K_t^A + K_t^M + K_t^S = \left(1 + \frac{\alpha_A}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_A}\frac{L_t^A}{L_t^M} + \frac{\alpha_S}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_S}\frac{L_t^S}{L_t^M}\right)K_t^M \tag{58}$$

Hence,

$$K_t^A = \frac{\frac{\alpha_A}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_A}\frac{L_t^A}{L_t^M}}{1+\frac{\alpha_A}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_A}\frac{L_t^A}{L_t^M}+\frac{\alpha_S}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_S}\frac{L_t^S}{L_t^M}}K_t \tag{59}$$

$$K_t^S = \frac{\frac{\alpha_S}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_S}\frac{L_t^S}{L_t^M}}{1+\frac{\alpha_A}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_A}\frac{L_t^A}{L_t^M}+\frac{\alpha_S}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_S}\frac{L_t^S}{L_t^M}}K_t \tag{60}$$

$$K_t^M = \frac{1}{1+\frac{\alpha_A}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_A}\frac{L_t^A}{L_t^M}+\frac{\alpha_S}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_S}\frac{L_t^S}{L_t^M}}K_t \tag{61}$$

Therefore,

$$A_t^A = \frac{Y_t^A}{(K_t^A)^{\alpha_A}(L_t^A)^{1-\alpha_A}} \tag{62}$$

$$A_t^S = \frac{Y_t^S}{(K_t^S)^{\alpha_S}(L_t^S)^{1-\alpha_S}} \tag{63}$$

$$A_t^M = \frac{Y_t^M}{(K_t^M)^{\alpha_M}(L_t^M)^{1-\alpha_M}} \tag{64}$$

$$\gamma_t^A = \frac{Y_t^A}{N_t}\left[1-\frac{\eta_A}{\eta_M}\frac{1-\alpha_A}{1-\alpha_M}\frac{L_t^M}{L_t^A}\frac{Y_t^M-K_{t+1}+(1-\delta)K_t}{Y_t^M}\right] \tag{65}$$

$$\gamma_t^S = \frac{Y_t^S}{N_t}\left(1-\frac{\eta_S}{\eta_M}\frac{1-\alpha_S}{1-\alpha_M}\frac{L_t^M}{L_t^S}\frac{Y_t^M-K_{t+1}+(1-\delta)K_t}{Y_t^M}\right) \tag{66}$$

Use

$$\frac{\eta_A}{\eta_M}\frac{Y_t^M-K_{t+1}+(1-\delta)K_t}{\frac{1-\alpha_M}{1-\alpha_A}\frac{L_t^A}{L_t^M}\frac{Y_t^M}{Y_t^A}} = Y_t^A - N_t\gamma_t^A \tag{67}$$

$$\frac{\eta_S}{\eta_M}\frac{Y_t^M-K_{t+1}+(1-\delta)K_t}{\frac{1-\alpha_M}{1-\alpha_S}\frac{L_t^S}{L_t^M}\frac{Y_t^M}{Y_t^S}} = Y_t^S - N_t\gamma_t^S \tag{68}$$

Substitute into Euler equation to solve for capital:

$$1 = E_t\left[\begin{array}{c}\left(\left(\frac{L_t^A}{L_{t+1}^A}\frac{L_{t+1}^M}{L_t^M}\frac{Y_t^M}{Y_{t+1}^M}\frac{Y_{t+1}^A}{Y_t^A}\right)^{\eta_A}\left(\frac{L_t^S}{L_{t+1}^S}\frac{L_{t+1}^M}{L_t^M}\frac{Y_t^M}{Y_{t+1}^M}\frac{Y_{t+1}^S}{Y_t^S}\right)^{\eta_S}\frac{Y_{t+1}^M-K_{t+2}+(1-\delta)K_{t+1}}{Y_t^M-K_{t+1}+(1-\delta)K_t}\frac{N_t}{N_{t+1}}\right)^{1-\theta} \\ *\frac{N_{t+1}}{N_t}\frac{Y_t^M-K_{t+1}+(1-\delta)K_t}{Y_{t+1}^M-K_{t+2}+(1-\delta)K_{t+1}}\beta\left(1-\delta+\alpha_M\frac{Y_{t+1}^M}{\frac{1}{1+\frac{\alpha_A}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_A}\frac{L_{t+1}^A}{L_{t+1}^M}+\frac{\alpha_S}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_S}\frac{L_{t+1}^S}{L_{t+1}^M}}K_{t+1}}\right)\end{array}\right] \tag{69}$$

$$1 = E_t\left[\begin{array}{c}\left(\left(\frac{L_t^A}{L_{t+1}^A}\frac{L_{t+1}^M}{L_t^M}\frac{Y_t^M}{Y_{t+1}^M}\frac{Y_{t+1}^A}{Y_t^A}\right)^{\eta_A}\left(\frac{L_t^S}{L_{t+1}^S}\frac{L_{t+1}^M}{L_t^M}\frac{Y_t^M}{Y_{t+1}^M}\frac{Y_{t+1}^S}{Y_t^S}\right)^{\eta_S}\right)^{1-\theta} \\ *\left(\frac{Y_{t+1}^M-K_{t+2}+(1-\delta)K_{t+1}}{Y_t^M-K_{t+1}+(1-\delta)K_t}\frac{N_t}{N_{t+1}}\right)^{-\theta}\beta\left(1-\delta+\alpha_M\left(1+\frac{\alpha_A}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_A}\frac{L_{t+1}^A}{L_{t+1}^M}+\frac{\alpha_S}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_S}\frac{L_{t+1}^S}{L_{t+1}^M}\right)\frac{Y_{t+1}^M}{K_{t+1}}\right)\end{array}\right] \tag{70}$$

$$\frac{B_{t+1}}{B_t} = \left(\left(\frac{L_t^A}{L_{t+1}^A}\frac{L_{t+1}^M}{L_t^M}\frac{Y_t^M}{Y_{t+1}^M}\frac{Y_{t+1}^A}{Y_t^A}\right)^{\eta_A}\left(\frac{L_t^S}{L_{t+1}^S}\frac{L_{t+1}^M}{L_t^M}\frac{Y_t^M}{Y_{t+1}^M}\frac{Y_{t+1}^S}{Y_t^S}\right)^{\eta_S}\right)^{1-\theta} \tag{71}$$

defines new variable B_t

$$B_t = \left(\frac{L_t^M}{L_t^A}\frac{Y_t^A}{Y_t^M}\right)^{\eta_A}\left(\frac{L_t^M}{L_t^S}\frac{Y_t^S}{Y_t^M}\right)^{\eta_S} \tag{72}$$

Denote

$$D_{t+1} = \alpha_M Y_{t+1}^M\left(1+\frac{\alpha_A}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_A}\frac{L_{t+1}^A}{L_{t+1}^M}+\frac{\alpha_S}{\alpha_M}\frac{1-\alpha_M}{1-\alpha_S}\frac{L_{t+1}^S}{L_{t+1}^M}\right) \tag{73}$$

Then, under perfect foresight:

$$1 = \beta\frac{B_{t+1}}{B_t}\left(\frac{Y_{t+1}^M - K_{t+2} + (1-\delta)K_{t+1}}{Y_t^M - K_{t+1} + (1-\delta)K_t}\frac{N_t}{N_{t+1}}\right)^{-\theta}\left(1-\delta+\frac{D_{t+1}}{K_{t+1}}\right) \tag{74}$$

VI.4. **Algorithm.** To summarize, the algorithm is the following:
Calibrate: $\beta, \delta, \alpha_A, \alpha_M, \alpha_S, \eta_A, \eta_M, \eta_S, \theta$.
Take quantity data: $Y_t^A, Y_t^M, Y_t^S, L_t^A, L_t^M, L_t^S, L_t, N_t$.
Labor data has to be comparable across sectors!
Compute:

$$B_t = \left(\frac{Y_t^A}{L_t^A}\right)^{\eta_A}\left(\frac{Y_t^S}{L_t^S}\right)^{\eta_S}\left(\frac{L_t^M}{Y_t^M}\right)^{\eta_A+\eta_S} \tag{75}$$

Compute

$$D_t = \alpha_M Y_t^M\left(1+\frac{\alpha_A}{1-\alpha_A}\frac{1-\alpha_M}{\alpha_M}\frac{L_t^A}{L_t^M}+\frac{\alpha_S}{1-\alpha_S}\frac{1-\alpha_M}{\alpha_M}\frac{L_t^S}{L_t^M}\right) \tag{76}$$

Project some values B_{T+1} and D_{T+1}, fix some values of K_0 and K_{T+1}.
Solve a system of T equations in $K_{1\ldots T}$ given K_0 and K_{T+1}:

$$1 = \beta\frac{B_{t+1}}{B_t}\left(\frac{Y_{t+1}^M - K_{t+2} + (1-\delta)K_{t+1}}{Y_t^M - K_{t+1} + (1-\delta)K_t}\frac{N_t}{N_{t+1}}\right)^{-\theta}\left(1-\delta+\frac{D_{t+1}}{K_{t+1}}\right) \tag{77}$$

Compute capital shares:

$$K_t^A = \frac{\frac{\alpha_A}{1-\alpha_A}L_t^A}{\frac{\alpha_A}{1-\alpha_A}L_t^A+\frac{\alpha_M}{1-\alpha_M}L_t^M+\frac{\alpha_S}{1-\alpha_S}L_t^S}K_t \tag{78}$$

$$K_t^S = \frac{\frac{\alpha_S}{1-\alpha_S}L_t^S}{\frac{\alpha_A}{1-\alpha_A}L_t^A+\frac{\alpha_M}{1-\alpha_M}L_t^M+\frac{\alpha_S}{1-\alpha_S}L_t^S}K_t \tag{79}$$

$$K_t^M = \frac{\frac{\alpha_M}{1-\alpha_M}L_t^M}{\frac{\alpha_A}{1-\alpha_A}L_t^A+\frac{\alpha_M}{1-\alpha_M}L_t^M+\frac{\alpha_S}{1-\alpha_S}L_t^S}K_t \tag{80}$$

Compute technology and preferences:

$$A_t^A = \frac{Y_t^A}{\left(K_t^A\right)^{\alpha_A}\left(L_t^A\right)^{1-\alpha_A}} \tag{81}$$

$$A_t^S = \frac{Y_t^S}{\left(K_t^S\right)^{\alpha_S}\left(L_t^S\right)^{1-\alpha_S}} \tag{82}$$

$$A_t^M = \frac{Y_t^M}{\left(K_t^M\right)^{\alpha_M}\left(L_t^M\right)^{1-\alpha_M}} \tag{83}$$

$$\gamma_t^A = \frac{Y_t^A}{N_t}\left[1 - \frac{\eta_A}{\eta_M}\frac{1-\alpha_A}{1-\alpha_M}\frac{L_t^M}{L_t^A}\frac{Y_t^M - K_{t+1} + (1-\delta)K_t}{Y_t^M}\right] \tag{84}$$

$$\gamma_t^S = \frac{Y_t^S}{N_t}\left(1 - \frac{\eta_S}{\eta_M}\frac{1-\alpha_S}{1-\alpha_M}\frac{L_t^M}{L_t^S}\frac{Y_t^M - K_{t+1} + (1-\delta)K_t}{Y_t^M}\right) \tag{85}$$

VI.5. **Counterfactuals.** Assume path of capital does not change. Fix K_t.
Substitute in:

$$c_t^A = Y_t^A/N_t \qquad c_t^M = \frac{Y_t^M - K_{t+1} + (1-\delta)K_t}{N_t} \tag{86}$$

$$c_t^S = Y_t^S/N_t \tag{87}$$

$$p_t^A = \frac{\eta_A}{\eta_M}\frac{Y_t^M - K_{t+1} + (1-\delta)K_t}{Y_t^A - \gamma_t^A N_t} \tag{88}$$

$$p_t^S = \frac{\eta_S}{\eta_M}\frac{Y_t^M - K_{t+1} + (1-\delta)K_t}{Y_t^S - \gamma_t^S N_t} \tag{89}$$

$$c_t = \left(c_t^A - \gamma_t^A\right)^{\eta_A}\left(c_t^M\right)^{\eta_M}\left(c_t^S - \gamma_t^S\right)^{\eta_S} \tag{90}$$

$$r_t = \alpha_M \frac{Y_t^M}{K_t^M} \tag{91}$$

$$w_t = (1-\alpha_M)\frac{Y_t^M}{L_t^M} \tag{92}$$

18 prices and allocations: $\left\{K_t^{A,M,S}, L_t^{A,M,S}, Y_t^{A,M,S}, c_t^{A,M,S}, K_{t+1}, c_t, p_t^{A,S}, r_t, w_t\right\}$
with $\left\{A_t^{A,M,S}, \gamma_t^{A,S}, L_t, N_t\right\}$ exogenously given
Here is the system to be solved:

$$(1) \qquad \frac{K_t^A}{K_t^M} = \frac{\eta_A}{\eta_M}\frac{\alpha_A}{\alpha_M}\frac{Y_t^A}{Y_t^A - \gamma_t^A N_t}\frac{Y_t^M - K_{t+1} + (1-\delta)K_t}{Y_t^M} \tag{93}$$

$$(2) \qquad \frac{K_t^S}{K_t^M} = \frac{\eta_S}{\eta_M}\frac{\alpha_S}{\alpha_M}\frac{Y_t^S}{Y_t^S - \gamma_t^S N_t}\frac{Y_t^M - K_{t+1} + (1-\delta)K_t}{Y_t^M} \tag{94}$$

$$(3) \qquad \frac{L_t^A}{L_t^M} = \frac{\eta_A}{\eta_M}\frac{1-\alpha_A}{1-\alpha_M}\frac{Y_t^A}{Y_t^A - \gamma_t^A N_t}\frac{Y_t^M - K_{t+1} + (1-\delta)K_t}{Y_t^M} \tag{95}$$

$$(4) \qquad \frac{L_t^S}{L_t^M} = \frac{\eta_S}{\eta_M}\frac{1-\alpha_S}{1-\alpha_M}\frac{Y_t^S}{Y_t^S - \gamma_t^S N_t}\frac{Y_t^M - K_{t+1} + (1-\delta)K_t}{Y_t^M} \tag{96}$$

$$(5) \qquad K_t = K_t^A + K_t^M + K_t^S \tag{97}$$

$$(6) \qquad L_t = L_t^A + L_t^M + L_t^S \tag{98}$$

$$(7) \qquad Y_t^M = A_t^M\left(K_t\right)^{\alpha_M}\left(L_t\right)^{1-\alpha_M} \tag{99}$$

$$(8) \qquad Y_t^A = A_t^A\left(K_t^A\right)^{\alpha_A}\left(L_t^A\right)^{1-\alpha_A} \tag{100}$$

$$(9) \qquad Y_t^S = A_t^S \left(K_t^S\right)^{\alpha_S} \left(L_t^S\right)^{1-\alpha_S} \tag{101}$$

We know: $\left\{A_t^{A,M,S}, \gamma_t^{A,S}, L_t, N_t, K_t\right\}$ We solve for: $\left\{Y_t^{A,M,S}, K_t^{A,M,S}, L_t^{A,M,S}\right\}$

VI.6. Elasticities.

$$\max c_t = \left(c_t^A - \gamma_t^A\right)^{\eta_A} \left(c_t^M\right)^{\eta_M} \left(c_t^S - \gamma_t^S\right)^{\eta_S} \qquad s.t. \qquad p_t^A c_t^A + c_t^M + p_t^S c_t^S = I \tag{102}$$

FOC:

$$\frac{\eta_A c_t}{\left(c_t^A - \gamma_t^A\right)} = p_t^A \lambda_t \tag{103}$$

$$\frac{\eta_M c_t}{c_t^M} = \lambda_t \tag{104}$$

$$\frac{\eta_S c_t}{\left(c_t^S - \gamma_t^S\right)} = p_t^S \lambda_t \tag{105}$$

Therefore,

$$p_t^A c_t^A = \frac{\eta_A}{\eta_M} c_t^M + p_t^A \gamma_t^A, \tag{106}$$

$$p_t^S c_t^S = \frac{\eta_S}{\eta_M} c_t^M + p_t^S \gamma_t^S \tag{107}$$

$$I = p_t^A c_t^A + c_t^M + p_t^S c_t^S = \frac{c_t^M}{\eta_M} + p_t^A \gamma_t^A + p_t^S \gamma_t^S \tag{108}$$

$$c_t^M = \eta_M \left(I - p_t^A \gamma_t^A - p_t^S \gamma_t^S\right) \tag{109}$$

$$c_t^A = \frac{\eta_A}{p_t^A} \left(I - p_t^A \gamma_t^A - p_t^S \gamma_t^S\right) + \gamma_t^A \tag{110}$$

$$c_t^S = \frac{\eta_S}{p_t^S} \left(I - p_t^A \gamma_t^A - p_t^S \gamma_t^S\right) + \gamma_t^S \tag{111}$$

$$\frac{p_t^A \left(c_t^A - \gamma_t^A\right)}{\eta_A} + p_t^A \gamma_t^A + p_t^S \gamma_t^S = I = \frac{p_t^S \left(c_t^S - \gamma_t^S\right)}{\eta_S} + p_t^A \gamma_t^A + p_t^S \gamma_t^S \tag{112}$$

$$\frac{\partial c_t^M}{\partial I} \frac{I}{c_t^M} = \eta_M \frac{\frac{c_t^M}{\eta_M} + p_t^A \gamma_t^A + p_t^S \gamma_t^S}{c_t^M} = 1 + \eta_M \frac{p_t^A \gamma_t^A + p_t^S \gamma_t^S}{c_t^M} \tag{113}$$

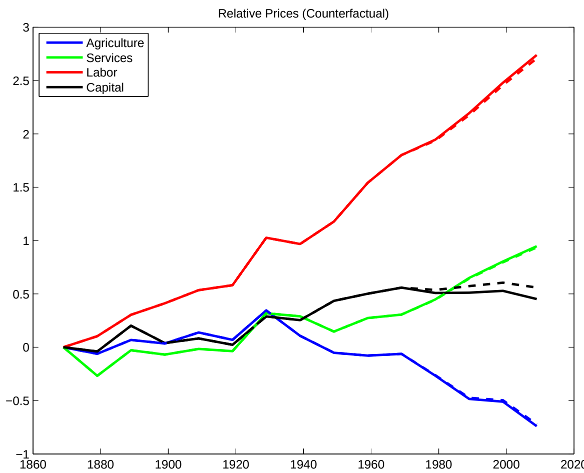
$$\frac{\partial c_t^A}{\partial I} \frac{I}{c_t^A} = \frac{\eta_A}{p_t^A} \frac{\frac{\left(c_t^A - \gamma_t^A\right) p_t^A}{\eta_A} + p_t^A \gamma_t^A + p_t^S \gamma_t^S}{c_t^A} = 1 + \frac{\left(\eta_A - 1\right) \gamma_t^A + \eta_A \frac{p_t^S}{p_t^A} \gamma_t^S}{c_t^A} \tag{114}$$

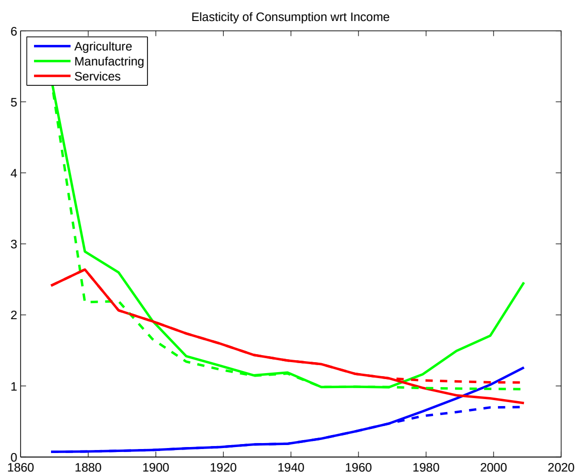
$$\frac{\partial c_t^S}{\partial I} \frac{I}{c_t^S} = \frac{\eta_S}{p_t^S} \frac{\frac{\left(c_t^S - \gamma_t^S\right) p_t^S}{\eta_S} + p_t^A \gamma_t^A + p_t^S \gamma_t^S}{c_t^S} = 1 + \frac{\left(\eta_S - 1\right) \gamma_t^S + \eta_S \frac{p_t^A}{p_t^S} \gamma_t^A}{c_t^S} \tag{115}$$

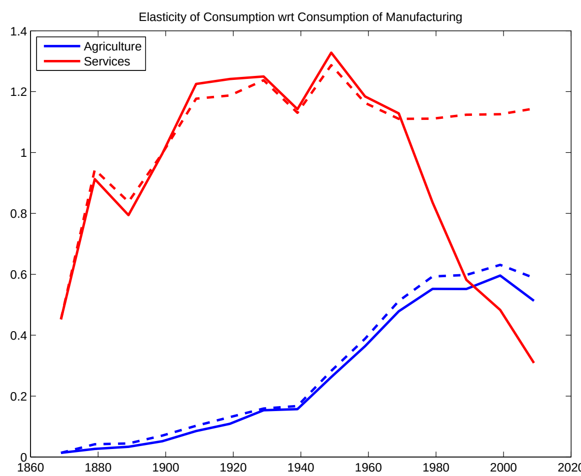
$$\frac{\partial c_t^A}{\partial c_t^M} \frac{c_t^M}{c_t^A} = \frac{1}{1 + \frac{\eta_M}{\eta_A} \frac{p_t^A \gamma_t^A}{c_t^M}} \tag{116}$$

$$(5) \frac{\partial c_t^S}{\partial c_t^M} \frac{c_t^M}{c_t^S} = \frac{1}{1 + \frac{\eta_M}{\eta_S} \frac{p_t^S \gamma_t^S}{c_t^M}} \tag{117}$$

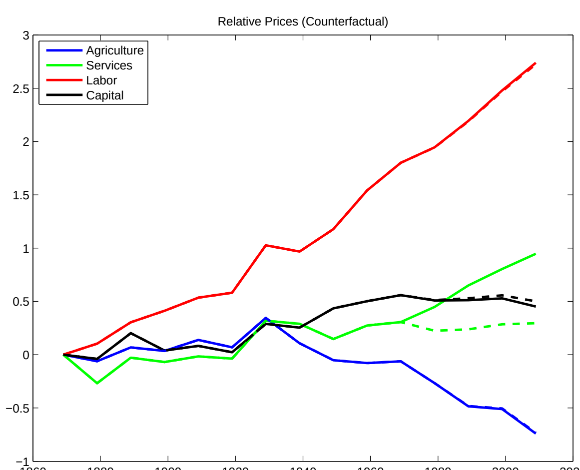
VI.7. **Extra Graphs.** Counterfactual 1:

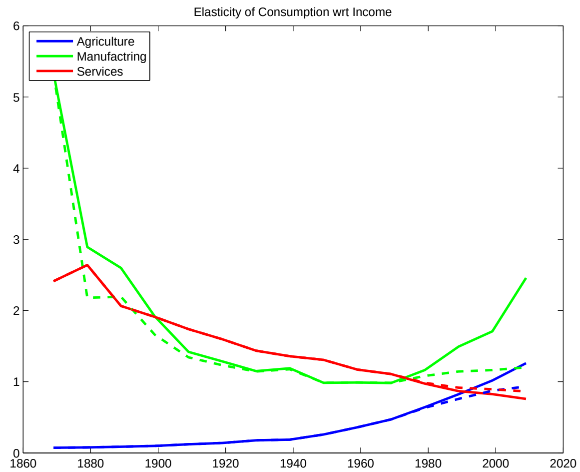


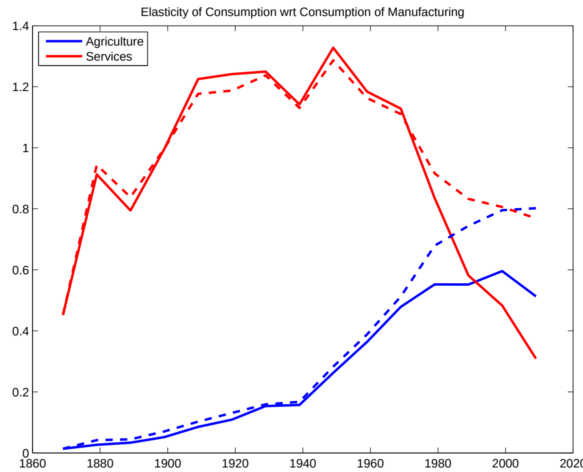




Counterfactual 2:




Elasticity of Consumption wrt Income
Agriculture
Manufactring
Services
0
1
2
3
4
5
6
1860
1880
1900
1920
1940
1960
1980
2000
2020


Elasticity of Consumption wrt Consumption of Manufacturing
Agriculture
Services
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1860
1880
1900
1920
1940
1960
1980
2000
2020

UCLA