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## Business Cycles in a Snow White Economy

The economy features Snow White and 7 Dwarves. Snow White uses an exercise bicycle to lose weight and at the same time produces  $E$  hours of electricity. Snow White uses  $m$  hours of electricity to read a book and buys  $q$  kg of candy. She loves candies and dislikes exercising. Snow White's preferences are quadratic in candies and linear in exercising time:

$$\begin{aligned} u(q, m) &= 6q - \frac{q^2}{2} - E \rightarrow \max_{q, m-E} \\ \text{s.t.} \quad & pq = wE + \sum_{i=1}^N \pi_i \\ \text{FOC:} \quad & 6 - q - \frac{p}{w} = 0 \end{aligned}$$

Here  $p$  is the price of candy, and  $w$  is the price of electricity per hour. It follows, that Snow White's demand is determined by a linear function:

$$q(p) = 6 - \frac{p}{w}$$

To produce a candy each Dwarf needs an original idea how to do it. Every night unemployed Dwarves smoke pipes and a new idea comes to one of them. That Dwarf uses his idea to build a candy-making machine and starts operating it. The machine is costless.

The candy-making machine falls apart slowly, becoming less and less efficient. When the machine is new, each kilogram of candy requires 1 hour of Dwarf operating time. The cost of a candy increases 10 percent every day, so a Dwarf needs 10% more time to produce a kilogram of candy.

There is a dark cave next to the house which contains gems. Dwarves love gems. They need one hour of light time in the cave each to dig out a gem. Digging together is not productive. The only reason a Dwarf produces candy is to earn the wage, which allows him to buy one hour of electricity. So he works exactly one hour (supplies one hour inelastically). If a Dwarf does not get a full hour of digging, he does not get the gem, so he becomes really pissed off and breaks down his candy-making machine. Dwarves which don't have a candy making machine do not get any electricity and do not dig gems.

Every evening all candies are mixed together and sold to Snow White. Remember that Snow White's demand is determined by a linear function:  $q(p) = 6 - \frac{p}{w}$ . If there are  $N$  Dwarves producing candy at the beginning of period  $t$  then together they face inverse demand  $\frac{p(q)}{w} = 6 - q$ .

Dwarf  $i$ 's production of candy:

$$q_i = z_i$$

Dwarf  $i$ 's profit:

$$\pi_i = p(q) z_i - w$$

Total quantity:

$$q = \sum_{i=1}^N z_i$$

Equilibrium price:

$$p = (6 - \sum_{i=1}^N z_i) w$$

If Dwarf i's profits are positive, he gives them to Snow White. If profits are negative, that means the Dwarf will not have a full hour of time to spend in the cave at the end of the day. Electricity suddenly goes out, he becomes angry, destroys his machine and starts smoking pipe again.

As mentioned before, productivity is determined by

$$z_i = 0.9^{i-1}$$

Total output is equal to

$$\sum_{i=1}^N z_i = \frac{1-0.9^N}{1-0.9}$$

Now consider an example. In one of the days 5 Dwarves have machines and are producing candy. Then

$$q = \left. \frac{1-0.9^N}{1-0.9} \right|_{N=5} = 4.10 \text{ kg}$$

are produced, the price of a kilogram of candy is

$$\frac{p}{w} = 6 - q = 1.90 \text{ h/kg}$$

All five Dwarves have profits

$$\begin{aligned} \pi_i/w &= p(q) z_i/w - 1 = 1.31 * 0.9^{i-1} - 1 \\ &= [0.90, 0.71, 0.54, 0.39, 0.25] \text{ h} \end{aligned}$$

and continue producing next period.

Next period the sixth Dwarf comes up with a new idea, then

$$q = \left. \frac{1-0.9^N}{1-0.9} \right|_{N=6} = 4.69 \text{ kg}$$

of sweets are produced, the price is

$$\frac{p}{w} = 6 - q = 1.31 \text{ h/kg}$$

and profits are now

$$\begin{aligned} \pi_i/w &= p(q) z_i/w - 1 = 1.31 * 0.9^{i-1} - 1 \\ &= [0.31, 0.18, 0.06, -0.04, -0.13, -0.22] \text{ h} \end{aligned}$$

This means, that three Dwarves will end up in the darkness without spending enough time digging gems. So they become angry and break down their machines.

They have no incentive to wait for the next period with the old machines in hands, because under the existing circumstances of high competition they will not get positive profits ever again in the future. The important assumption here is that they cannot coordinate on who digs gems and keeps the machine till next period and who doesn't.

Snow White will blame Dwarves for "animal spirits" and "the economy" for sudden crises of "overproduction" of candy. Business cycles dynamics are described in the graph. Markups are clearly counter-cyclical, labor and consumption are clearly procyclical. Business cycles arise endogenously, in absense of exogenous shocks.

