A Model of the Small Open Economy — Anton Cheremukhin

November 8, 2007

$$\max_{\{c_t,a_{t+1},k_t\}} E_t\Sigma_{t=0}^{\infty}\beta^t u(c_t,n_t)$$

$$\text{s.t.} \quad u\left(c_t,n_t\right)=\frac{1}{1-\sigma}\left(c_t^{\psi}\left(1-n_t\right)^{1-\psi}\right)^{1-\sigma} \quad or \quad u\left(c_t,n_t\right)=\frac{1}{1-\sigma}\left(c_t-\frac{\psi_0}{\psi}n_t^{\psi}\right)^{1-\sigma}$$

$$\text{s.t.} \quad c_t=\left[\gamma\left(c_t^T\right)^{\rho}+\left(1-\gamma\right)\left(c_t^N\right)^{\rho}\right]^{1/\rho} \quad \text{s.t.} \quad c_t^N=A^N z_t^N\left(n_t^N\right)^{1-\eta}$$

$$\text{s.t.} \quad c_t^T+i_t+\frac{\omega}{2}\frac{i_t^2}{k_t}+a_{t+1}=A^T z_t^T\left(k_t^T\right)^{\alpha}\left(n_t^T\right)^{1-\alpha}-G_t*y_t^T+\left(1+r+\varepsilon_t\right)a_t$$

$$\text{s.t.} \quad i_t=k_{t+1}-\left(1-\delta\right)k_t \quad \text{s.t.} \quad n_t^T+n_t^T=n_t \quad \text{s.t.} \quad \varepsilon_t=\varepsilon\left(e^{a_{ss}-a_t}-1\right)$$

Lagrangian:

$$\max E_t\Sigma_{t=0}^{\infty}\beta^t\left\{\begin{array}{c} u(c_t\left(c_t^T,c_t^N\right),n_t)+\mu_t\left(n_t-n_t^T-n_t^N\right)+\nu_t\left(z_t^N\left(n_t^N\right)^{1-\eta}-c_t^N\right)+ \\ \lambda_t\left(\begin{array}{c} z_t^T\left(k_t\right)^{\alpha}\left(n_t^T\right)^{1-\alpha}-G_t*y_t^T-a_{t+1}+\left(1+r+\varepsilon_t\right)a_t-c_t^T-i_t-\frac{\omega}{2}\frac{i_t^2}{k_t}+ \\ +q_t\left(i_t-k_{t+1}+\left(1-\delta\right)k_t\right) \end{array}\right) \end{array}\right\}$$

$\text{FOC}_{c_t^T}: \quad u_{c,t}*c_{T,t}=\lambda_t$

$\text{FOC}_{c_t^N}: \quad u_{c,t}*c_{N,t}=\nu_t$

$\text{FOC}_{n_t}: \quad u_{n,t}=-\mu_t$

$\text{FOC}_{n_t^T}: \quad \mu_t=\lambda_t\left(1-\alpha\right)z_t^T\left(k_t^T\right)^{\alpha}\left(n_t^T\right)^{-\alpha}$

$\text{FOC}_{n_t^N}: \quad \mu_t=\nu_t\left(1-\eta\right)z_t^N\left(n_t^N\right)^{-\eta}$

$\text{FOC}_{a_{t+1}}: \quad \lambda_t=\beta\lambda_{t+1}\left(1+r+\varepsilon_{t+1}\right)$

$\text{FOC}_{i_t}: \quad q_t\lambda_t=\lambda_t\left(1+\omega\frac{i_t}{k_t}\right)$

$\text{FOC}_{k_{t+1}}: \quad q_t\lambda_t=\beta\lambda_{t+1}q_{t+1}\left(1-\delta\right)+\beta\lambda_{t+1}\left(\alpha z_{t+1}^T\left(k_{t+1}\right)^{\alpha-1}\left(n_{t+1}^T\right)^{1-\alpha}+\frac{\omega}{2}\frac{i_{t+1}^2}{k_{t+1}^2}\right)$

$\text{RC}_N: \quad c_t^N=z_t^N\left(n_t^N\right)^{1-\eta}$

$\text{RC}_I: \quad i_t=k_{t+1}-\left(1-\delta\right)k_t$

$\text{RC}_n: \quad n_t^T+n_t^T=n_t$

$\text{RC}_T: \quad c_t^T+i_t+\frac{\omega}{2}\frac{i_t^2}{k_t}+a_{t+1}=z_t^T\left(k_t^T\right)^{\alpha}\left(n_t^T\right)^{1-\alpha}-G_t*y_t^T+\left(1+r+\varepsilon_t\right)a_t$

if $u\left(c_t,n_t\right)=\frac{1}{1-\sigma}\left(c_t^{\psi}\left(1-n_t\right)^{1-\psi}\right)^{1-\sigma}$ then

$u_c=\psi c_t^{\psi-1}\left(1-n_t\right)^{1-\psi}\left(c_t^{\psi}\left(1-n_t\right)^{1-\psi}\right)^{-\sigma}$

$u_n=-\left(1-\psi\right)c_t^{\psi}\left(1-n_t\right)^{-\psi}\left(c_t^{\psi}\left(1-n_t\right)^{1-\psi}\right)^{-\sigma}$

if $u\left(c_t,n_t\right)=\frac{1}{1-\sigma}\left(c_t-\frac{\psi_0}{\psi}n_t^{\psi}\right)^{1-\sigma}$ then

$u_c=\left(c_t-\frac{\psi_0}{\psi}n_t^{\psi}\right)^{-\sigma}$

$u_n=-\psi_0 n_t^{\psi-1}\left(c_t-\frac{\psi_0}{\psi}n_t^{\psi}\right)^{-\sigma}$

if $c_t=\left[\gamma\left(c_t^T\right)^{\rho}+\left(1-\gamma\right)\left(c_t^N\right)^{\rho}\right]^{\frac{1}{\rho}}$ then

$c_{T,t}=\gamma\left(c_t^T\right)^{\rho-1}\left[\gamma\left(c_t^T\right)^{\rho}+\left(1-\gamma\right)\left(c_t^N\right)^{\rho}\right]^{\frac{1}{\rho}-1}$

$c_{N,t}=\left(1-\gamma\right)\left(c_t^N\right)^{\rho-1}\left[\gamma\left(c_t^T\right)^{\rho}+\left(1-\gamma\right)\left(c_t^N\right)^{\rho}\right]^{\frac{1}{\rho}-1}$

The system is equivalent to:

AGG: $c_t = \left[\gamma \left(c_t^T\right)^{\rho} + (1-\gamma)\left(c_t^N\right)^{\rho}\right]^{\frac{1}{\rho}}$

FOC$_{c_t^T}$: $\left(c_t - \frac{\psi_0}{\psi} n_t^{\psi}\right)^{-\sigma} * \gamma \left(\frac{c_t}{c_t^T}\right)^{1-\rho} = \lambda_t$

FOC$_{c_t^N}$: $\left(c_t - \frac{\psi_0}{\psi} n_t^{\psi}\right)^{-\sigma} * (1-\gamma) \left(\frac{c_t}{c_t^N}\right)^{1-\rho} = \nu_t$

FOC$_{n_t}$: $\psi_0 n_t^{\psi-1} \left(c_t - \frac{\psi_0}{\psi} n_t^{\psi}\right)^{-\sigma} = \mu_t$

FOC$_{n_t^T}$: $\mu_t = \lambda_t (1-\alpha) z_t^T \left(k_t^T\right)^{\alpha} \left(n_t^T\right)^{-\alpha}$

FOC$_{n_t^N}$: $\mu_t = \nu_t (1-\eta) z_t^N \left(n_t^N\right)^{-\eta}$

FOC$_{a_{t+1}}$: $\lambda_t = \beta \lambda_{t+1} \left(1 + r + \varepsilon\left(e^{a_0 - a_{t+1}} - 1\right)\right)$

FOC$_{k_{t+1}}$: $q_t \lambda_t = \beta \lambda_{t+1} q_{t+1} (1-\delta) + \beta \lambda_{t+1} \left(\alpha z_{t+1}^T \left(k_{t+1}\right)^{\alpha-1} \left(n_{t+1}^T\right)^{1-\alpha} + \frac{\omega}{2} \frac{i_{t+1}^2}{k_{t+1}^2}\right)$

FOC$_{i_t}$: $q_t \lambda_t = \lambda_t \left(1 + \omega \frac{i_t}{k_t}\right)$

RC$_N$: $c_t^N = z_t^N \left(n_t^N\right)^{1-\eta}$

RC$_T$: $c_t^T + i_t + \frac{\omega}{2} \frac{i_t^2}{k_t} + a_{t+1} = z_t^T \left(k_t^T\right)^{\alpha} \left(n_t^T\right)^{1-\alpha} - G_t * y_t^T + \left(1 + r + \varepsilon\left(e^{a_0 - a_t} - 1\right)\right) a_t$

RC$_I$: $i_t = k_{t+1} - (1-\delta) k_t$

RC$_n$: $n_t^T + n_t^T = n_t$

Z$_T$: $\ln z_{t+1}^T = (1-\rho_T) \ln z_0^T + \rho_T \ln z_t^T + e_{T,t+1}$

Z$_N$: $\ln z_{t+1}^N = (1-\rho_N) \ln z_0^N + \rho_N \ln z_t^N + e_{N,t+1}$

T$_t$: $\ln G_{t+1} = (1-\rho_G) \ln G_0 + \rho_G \ln G_t + e_{G,t+1}$

TB: $TB_t = \left(a_{t+1} - a_t (1+r)\right) / y_t^T + e_{1,t}$ (for measurement purposes)

Now I simplify the model a little bit:

AGG: $c_t = \left[\gamma \left(c_t^T\right)^{\rho} + (1-\gamma)\left(c_t^N\right)^{\rho}\right]^{\frac{1}{\rho}}$

FOC$_{n_t^T}$: $\psi_0 n_t^{\psi-1} = \gamma \left(\frac{c_t}{c_t^T}\right)^{1-\rho} (1-\alpha) z_t^T \left(k_t^T\right)^{\alpha} \left(n_t^T\right)^{-\alpha}$

FOC$_{n_t^N}$: $\psi_0 n_t^{\psi-1} = (1-\gamma) \left(\frac{c_t}{c_t^N}\right)^{1-\rho} (1-\eta) z_t^N \left(n_t^N\right)^{-\eta}$

FOC$_{a_{t+1}}$: $\left(c_t - \frac{\psi_0}{\psi} n_t^{\psi}\right)^{-\sigma} \left(\frac{c_t}{c_t^T}\right)^{1-\rho} = \left(c_{t+1} - \frac{\psi_0}{\psi} n_{t+1}^{\psi}\right)^{-\sigma} \left(\frac{c_{t+1}}{c_{t+1}^T}\right)^{1-\rho} \left(1 + r + \varepsilon\left(e^{a_0 - a_{t+1}} - 1\right)\right) \frac{1}{1+r}$

FOC$_{k_{t+1}}$:

$\left(1 + r + \varepsilon\left(e^{a_0 - a_{t+1}} - 1\right)\right) \left(1 + \omega \frac{i_t}{k_t}\right) = \left(1 + \omega \frac{i_{t+1}}{k_{t+1}}\right)(1-\delta) + \alpha z_{t+1}^T \left(k_{t+1}\right)^{\alpha-1} \left(n_{t+1}^T\right)^{1-\alpha} + \frac{\omega}{2} \frac{i_{t+1}^2}{k_{t+1}^2}$

RC$_N$: $c_t^N = z_t^N \left(n_t^N\right)^{1-\eta}$

RC$_T$: $c_t^T + i_t + \frac{\omega}{2} \frac{i_t^2}{k_t} + a_{t+1} = z_t^T \left(k_t^T\right)^{\alpha} \left(n_t^T\right)^{1-\alpha} - G_t * y_t^T + \left(1 + r + \varepsilon\left(e^{a_0 - a_t} - 1\right)\right) a_t$

RC$_I$: $i_t = k_{t+1} - (1-\delta) k_t$

RC$_n$: $n_t^T + n_t^T = n_t$

Z$_T$: $\ln z_{t+1}^T = (1-\rho_T) \ln z_0^T + \rho_T \ln z_t^T + e_{T,t+1}$

Z$_N$: $\ln z_{t+1}^N = (1-\rho_N) \ln z_0^N + \rho_N \ln z_t^N + e_{N,t+1}$

T$_t$: $G_{t+1} = (1-\rho_G) G_0 + \rho_G G_t + e_{G,t+1}$

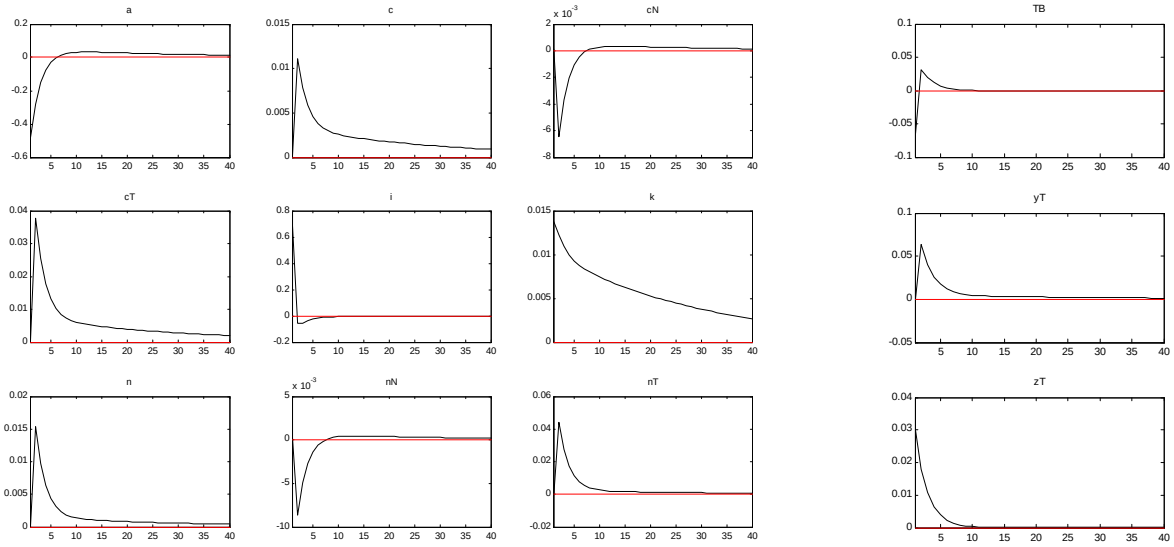
TB: $TB_t = \left(a_{t+1} - a_t (1+r)\right) / y_t^T + e_{1,t}$ (for measurement purposes)

Variables (chosen at time t): $\left\{c_t, c_t^T, c_t^N, n_t, n_t^T, n_t^N, i_t, k_{t+1}, a_{t+1}, z_{t+1}^T, z_{t+1}^N, G_{t+1}, TB_t\right\}$

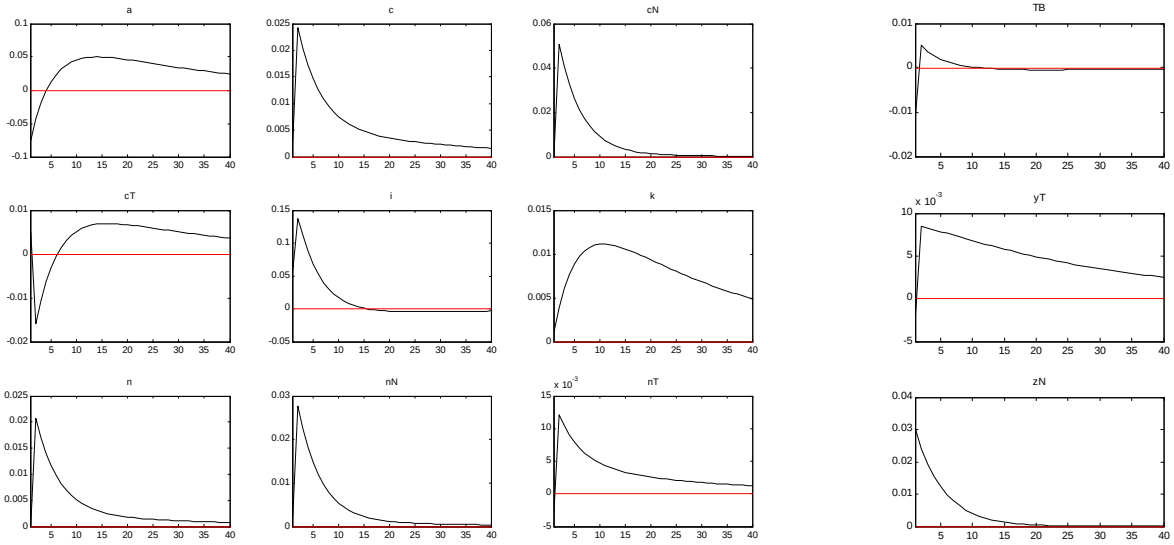
Parameters: $\left\{\gamma, \rho, \sigma, \psi, \psi_0, r, \alpha, \eta, \delta, \rho_T, \rho_N, \rho_G, \sigma_T, \sigma_N, \sigma_G, z_0^T, z_0^N, G, a_0, \omega, \varepsilon\right\}$ $\frac{1}{\beta} = 1 + r$

States (backward-looking): $\left\{k_t, a_t, z_t^T, z_t^N, G_t\right\}$ Forward-looking: $\left\{c_{t+1}^T, c_{t+1}, n_{t+1}, i_{t+1}, n_{t+1}^T\right\}$

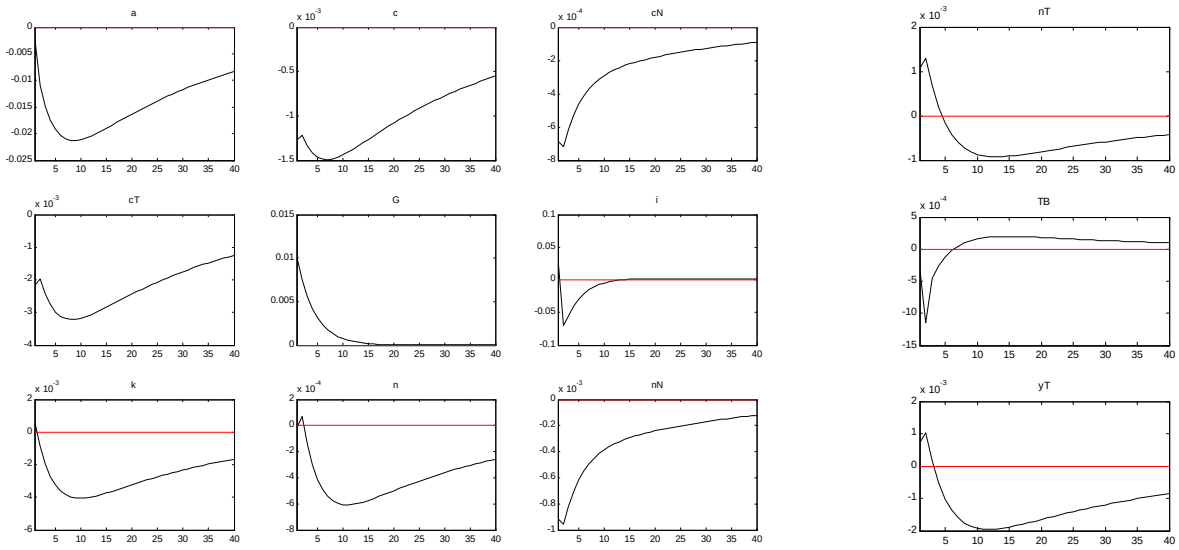
Response to e_T:



Response to e_N:



Response to e_G:



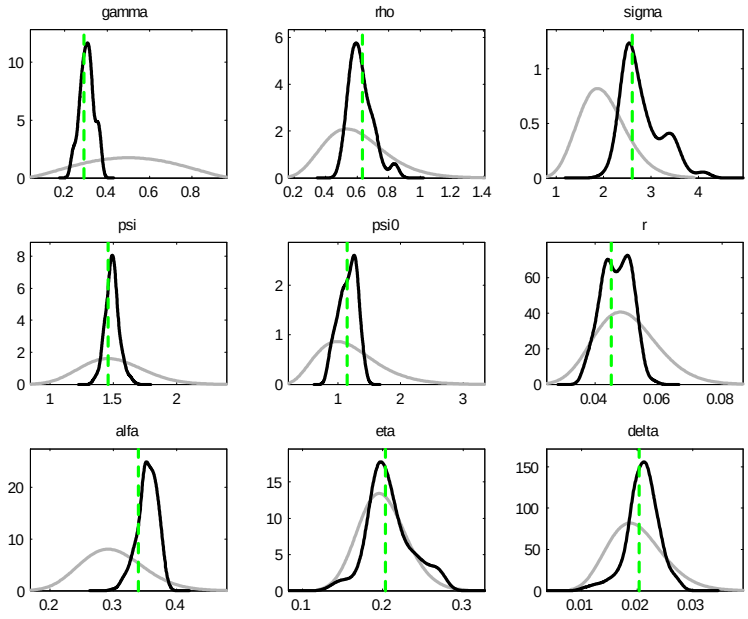
Posterior Mode

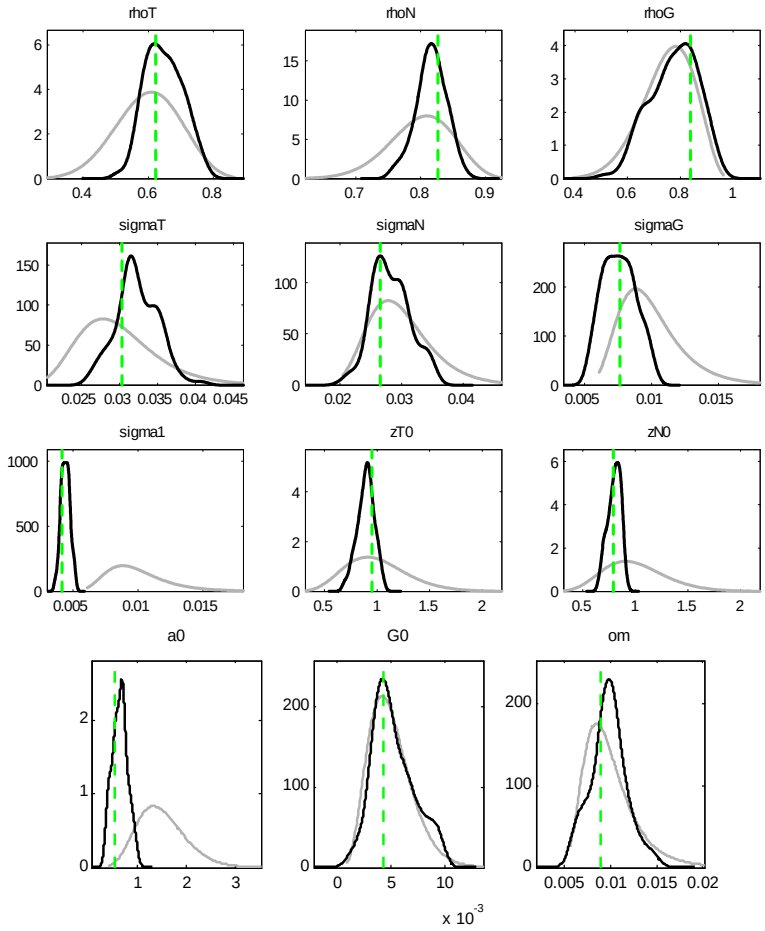
	Prior	True	Prior mean	Prior s.d.	Posterior mode	s.d.
γ	beta	0.30	0.500	0.2000	0.2912	0.0288
ρ	gamm	0.60	0.600	0.2000	0.6324	0.0730
σ_1	gamm	2.00	2.000	0.5000	2.6014	0.4447
ψ	gamm	1.60	1.500	0.2500	1.4599	0.0809
ψ_0	gamm	1.20	1.200	0.5000	1.1488	0.4449
r	gamm	0.05	0.050	0.0100	0.0450	0.0049
α	gamm	0.33	0.300	0.0500	0.3393	0.0225
η	gamm	0.25	0.200	0.0300	0.2034	0.0298
δ	gamm	0.02	0.020	0.0050	0.0204	0.0043
ρ_T	beta	0.60	0.600	0.1000	0.6223	0.0662
ρ_N	beta	0.80	0.800	0.0500	0.8266	0.0316
ρ_G	beta	0.75	0.750	0.1000	0.8379	0.0815
σ_T	invg	0.03	0.030	0.0050	0.0303	0.0023
σ_N	invg	0.03	0.030	0.0050	0.0265	0.0035
σ_G	invg	0.01	0.010	0.0025	0.0077	0.0012
σ_1	invg	.0025	0.010	0.0025	0.0042	0.0006
$Z_{T,0}$	gamm	1.00	1.000	0.3000	0.9496	0.1531
$Z_{N,0}$	gamm	1.00	1.000	0.3000	0.7906	0.1944
a_0	gamm	0.50	1.500	0.5000	0.5389	0.1386
G_0	gamm	0.20	0.005	0.0020	0.0043	0.0019
ω	invg	0.01	0.010	0.0030	0.0090	0.0021

Posterior Distribution

	Prior	True	Prior mean	Prior s.d.	Post. mean	HPD inf	HPD sup
γ	beta	0.30	0.500	0.2000	0.3049	0.2619	0.3678
ρ	gamm	0.60	0.600	0.2000	0.6223	0.5090	0.7313
σ_1	gamm	2.00	2.000	0.5000	2.7995	2.2722	3.5132
ψ	gamm	1.60	1.500	0.2500	1.4902	1.4082	1.5826
ψ_0	gamm	1.20	1.200	0.5000	1.1574	0.9075	1.3666
r	gamm	0.05	0.050	0.0100	0.0467	0.0397	0.0542
α	gamm	0.33	0.300	0.0500	0.3535	0.3284	0.3781
η	gamm	0.25	0.200	0.0300	0.2076	0.1733	0.2668
δ	gamm	0.02	0.020	0.0050	0.0209	0.0169	0.0262
ρ_T	beta	0.60	0.600	0.1000	0.6476	0.5597	0.7425
ρ_N	beta	0.80	0.800	0.0500	0.8161	0.7803	0.8559
ρ_G	beta	0.75	0.750	0.1000	0.7742	0.6335	0.9172
σ_T	invg	0.03	0.030	0.0050	0.0323	0.0271	0.0360
σ_N	invg	0.03	0.030	0.0050	0.0282	0.0241	0.0342
σ_G	invg	0.01	0.010	0.0025	0.0076	0.0057	0.0096
σ_1	invg	.0025	0.010	0.0025	0.0044	0.0039	0.0050
$Z_{T,0}$	gamm	1.00	1.000	0.3000	0.8919	0.7544	1.0141
$Z_{N,0}$	gamm	1.00	1.000	0.3000	0.7993	0.6852	0.8795
a_0	gamm	0.50	1.500	0.5000	0.6396	0.3763	0.8783
G_0	gamm	0.20	0.005	0.0020	0.0052	0.0030	0.0093
ω	invg	0.01	0.010	0.0030	0.0098	0.0063	0.0128

Posterior Distribution




rhoT
rhoN
rhoG
sigmaT
sigmaN
sigmaG
sigma1
zT0
zN0
a0
G0
om
x 10^-3