Take Home exam for Econ 232B Anton Cheremukhin

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Model

The motivation of this model is to combine different sources of shocks as potential explanations of the business cycle in a single simple model, which would allow to distinguish between the sources and to evaluate their relative significance.

The RBC model I use is a combination of different models with different sources of shocks. The main body of the model comes from the paper by Farmer and Hollenhorst (2006) which incorporate a two-sided matching technology into the process of accumulation of jobs and use Nash bargaining as a wage-setting mechanism. In addition to that I allow for neutral and investment-specific technological change as well as a preference shock. I borrow the shock structure from the paper by Fernandez-Villaverde and Rubio-Ramirez (2006) with the only difference that I do not allow variances to drift.

Let K_t denote capital stock, L_t - labor (stock of jobs, hours worked), U_t - time spent by a worker searching for a job, V_t - time spent by firms searching for workers, C_t - aggregate consumption of the representative worker (household), A_t - neutral technology used in the production of final output, T_t - investment-specific technology which is used to transform investment into capital, d_t - a shock to preferences.

Using this specification the planner's problem is to maximize welfare given resource constraints:

$$J_t = \max \Sigma_{t=s}^{\infty} \beta^{t-s} E_s \left[\exp(d_t) \log C_t - \chi \frac{L_t^{1+\gamma}}{1+\gamma} - b(U_t + V_t) \right]$$

Every period a fraction δ_L of jobs are lost, and new jobs are formed using a Cobb-Douglas matching technology:

$$L_t = (1 - \delta_L) L_{t-1} + B_t U_t^{\theta} V_t^{1-\theta}$$

Output is produced using a neutral technology and a standard Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

The fraction of output that is not consumed, is invested. Investment is transformed into capital using an investment-specific technology and replaces depreciated capital in steady-state.

$$K_{t+1} = (1 - \delta_K) K_t + V_t (Y_t - C_t)$$

Solution

To solve the model I write down the Lagrangian:

$$£ = \Sigma_{t=0}^{\infty}\beta^t E_s \left[\begin{array}{c} \exp\left(d_t\right)\frac{C_t^{1-\sigma}}{1-\sigma} - \chi\frac{L_t^{1+\gamma}}{1+\gamma} - b\left(U_t+V_t\right) + \\ \lambda_t\left(\left(1-\delta_K\right)K_t + T_tA_tK_t^{\alpha}L_t^{1-\alpha} - T_tC_t - K_{t+1}\right) + \\ \mu_t\left(\left(1-\delta_L\right)L_{t-1} + B_tU_t^{\theta}V_t^{1-\theta} - L_t\right) \end{array} \right] \rightarrow \max_{K_{t+1},L_t,U_t,V_t,C_t}$$

FOC$_{C_t}$: $\frac{\exp d_t}{C_t^{\sigma}} - V_t\lambda_t = 0$
FOC$_{L_t}$: $-\chi L_t^{\gamma} + \lambda_t\left(1-\alpha\right)\frac{T_tY_t}{L_t^{\alpha}} + \beta\mu_{t+1}\left(1-\delta_L\right) - \mu_t = 0$
FOC$_{K_{t+1}}$: $-\lambda_t + \beta\lambda_{t+1}\left(1-\delta_K + \alpha\frac{T_{t+1}Y_{t+1}}{K_{t+1}}\right) = 0$
FOC$_{U_t}$: $-b + \mu_t\theta\frac{M_t}{U_t} = 0$
FOC$_{V_t}$: $-b + \mu_t\left(1-\theta\right)\frac{M_t}{V_t} = 0$
The last two first-order conditions are equivalent to:

$$\frac{\theta}{U_t} = \frac{b}{\mu_tM_t} = \frac{1-\theta}{V_t}$$

Therefore the model can be simplified:

$$\frac{V_t}{U_t} = \frac{1-\theta}{\theta} = \varphi \qquad \frac{b}{\mu_t} = \frac{B_t(1-\theta)}{\varphi^{\theta}} \qquad \mu_t = \frac{b\varphi^{\theta}}{1-\theta}\frac{1}{B_t} = \frac{\omega}{B_t} \qquad \omega = \frac{b}{\theta^{\theta}(1-\theta)^{1-\theta}}.$$

Hence, the Lagrange multipliers are equal to:

$$\mu_t = \frac{\omega}{B_t} \qquad \lambda_t = \frac{\exp d_t}{C_t^{\sigma}T_t}$$

As a result, the system is equivalent to the following 6 equations:
(1) $\frac{\exp d_t}{C_t^{\sigma}V_t}\left(1-\alpha\right)\frac{T_tY_t}{L_t^{\alpha}} - \chi L_t^{\gamma} = \omega\left(\frac{1}{B_t} - \beta\frac{1-\delta_L}{B_{t+1}}\right)$
(2) $\frac{\exp d_t}{C_t^{\sigma}T_t} = \beta\frac{\exp d_{t+1}}{C_{t+1}^{\sigma}T_{t+1}}\left(1-\delta_K + \alpha\frac{T_{t+1}Y_{t+1}}{K_{t+1}}\right)$
(3) $K_{t+1} = \left(1-\delta_K\right)K_t + T_t\left(Y_t - C_t\right)$
(4) $L_t = \left(1-\delta_L\right)L_{t-1} + B_tU_t^{\theta}V_t^{1-\theta}$
(5) $\frac{\theta}{U_t} = \frac{1-\theta}{V_t}$
(6) $Y_t = A_tK_t^{\alpha}L_t^{1-\alpha}$
To close the model, equations 7-10 represent processes for exogenous shocks. Following the specification from the paper by Fernandez-Villaverde and Rubio-Ramirez (2006) I assume production and investment technologies to have unit roots:
(7) $\log A_t = \log a + \log A_{t-1} + \sigma_A\varepsilon_{At}$
(8) $\log T_t = \log\tau + \log T_{t-1} + \sigma_T\varepsilon_{Tt}$
I assume that preferences and the matching technology are stationary autoregressive processes:
(9) $d_t = \rho d_{t-1} + \sigma_d\varepsilon_{dt}$
(10) $\log B_t = \left(1-\rho_B\right)\log B + \rho_BB_{t-1} + \sigma_B\varepsilon_{Bt}$

Detrending

The model is non-stationary. Hence, to estimate it I will stationarize it by taking away the stochastic trend:

Call $Z_t = A_{t-1}^{\frac{1}{1-\alpha}} T_{t-1}^{\frac{\alpha}{1-\alpha}}$ $\quad a_t = \frac{A_t}{A_{t-1}}$ $\quad \tau_t = \frac{T_t}{T_{t-1}}$ $\quad c_t = \frac{C_t}{Z_t}$

$k_t = \frac{K_t}{Z_t T_{t-1}}$ $\quad z_{t-1} = \frac{Z_t}{Z_{t-1}} = \frac{A_{t-1}^{\frac{1}{1-\alpha}} T_{t-1}^{\frac{\alpha}{1-\alpha}}}{A_{t-2}^{\frac{1}{1-\alpha}} T_{t-2}^{\frac{\alpha}{1-\alpha}}} = a_{t-1}^{\frac{1}{1-\alpha}} \tau_{t-1}^{\frac{\alpha}{1-\alpha}}$

An additional implication of there existing a stationary version of the model is that risk-aversion σ ought to be set to unity. After substituting these into (1-10) the stationary version of the model is summarized by equations (1'-10'):

Labor-consumption decision:

(1') $\quad \frac{\exp d_t}{c_t} (1-\alpha) a_t \frac{k_t^\alpha}{L_t^\alpha} - \chi L_t^\gamma = \omega E_t \left(\frac{1}{B_t} - \beta \frac{1-\delta_L}{B_{t+1}} \right)$

Euler equation:

(2') $\quad 1 = \beta E_t \exp(d_{t+1} - d_t) \frac{c_t}{c_{t+1}} \frac{1}{z_t} \left(\frac{1-\delta_K}{\tau_{t+1}} + \alpha a_{t+1} \frac{L_{t+1}^{1-\alpha}}{k_{t+1}^{1-\alpha}} \right)$

Resource constraint for capital:

(3') $\quad k_{t+1} z_t = (1-\delta_K) \frac{k_t}{\tau_t} + a_t k_t^\alpha L_t^{1-\alpha} - c_t$

Resource constraint for labor:

(4') $\quad L_t = (1-\delta_L) L_{t-1} + B_t U_t^\theta V_t^{1-\theta}$

FOC for vacancies and labor effort:

(5') $\quad \theta V_t = (1-\theta) U_t$

Process for neutral technology:

(6') $\quad \log a_t = \log a + \sigma_A \varepsilon_{At}$

Process for investment-specific technology:

(7') $\quad \log \tau_t = \log \tau + \sigma_T \varepsilon_{Tt}$

Process for preference shock:

(8') $\quad d_t = \rho_d d_{t-1} + \sigma_d \varepsilon_{dt}$

Process for matching technology:

(9') $\quad \log B_t = (1-\rho_B) \log B_0 + \rho_B B_{t-1} + \sigma_B \varepsilon_{Bt}$

Aggregate trend:

(10') $\quad z_t^{1-\alpha} = a_t \tau_t^\alpha$

The state of the system is given by a vector: $[c_t, k_{t+1}, L_t, U_t, V_t, a_t, \tau_t, d_t, B_t, z_t]$

The parameter vector is: $\quad [\alpha, \chi, \omega, \gamma, \beta, \delta_K, \delta_L, \theta, \zeta, \xi, \rho_d, \rho_B, \sigma_A, \sigma_T, \sigma_d, \sigma_B]$

The absolute value of L should be calibrated to the data in steady-state.

Data and Measurement

In order to estimate this model, I write 4 measurement equations for hours worked, innovations to real GDP, real consumption and real investment. One can show that equations (11-14) hold:

(11) $\quad d \log Y_t^M = \log Y_t - \log Y_{t-1} = \log a_t + (1-\alpha) \log \frac{L_t}{L_{t-1}} + \alpha \left(\log \frac{k_t}{k_{t-1}} + \log z_{t-1} + \log \tau_{t-1} \right)$

(12) $\quad d \log C_t^M = \log C_t - \log C_{t-1} = \log \frac{c_t}{c_{t-1}} + \log z_{t-1}$

(13) $\quad d \log I_t^M = \log \frac{K_{t+1} - (1-\delta_K) K_t}{K_t - (1-\delta_K) K_{t-1}} = \log \frac{k_{t+1} z_t \tau_t - (1-\delta_K) k_t}{k_t z_{t-1} \tau_{t-1} - (1-\delta_K) k_{t-1}} + \log z_{t-1} + \log \tau_{t-1}$

(14) $\quad L_t = L_t \quad$ directly observed as hours worked

The data I use is for the United States. The data source is Federal Reserve Bank of St. Louis (http://research.stlouisfed.org/fred2).

As a measure of GDP I use "Real Gross Domestic Product", Quarterly.

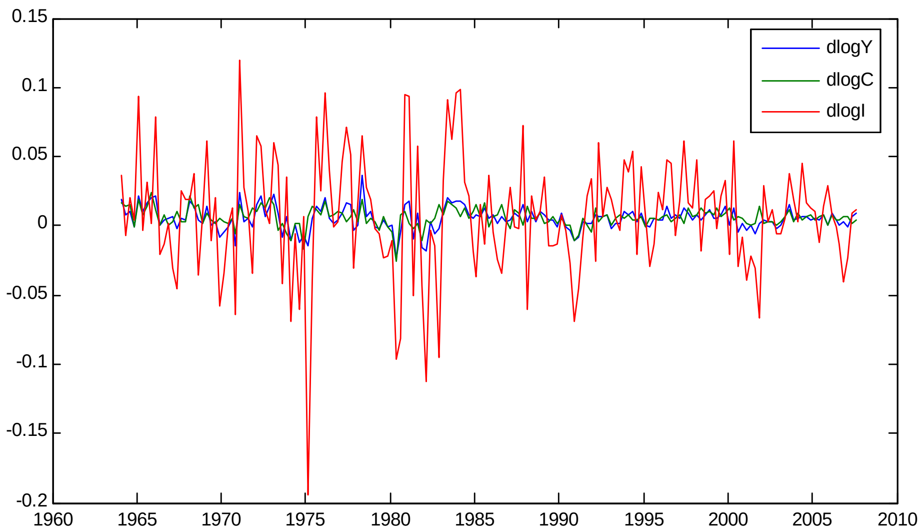
As a measure of Consumption I use "Real Personal Consumption Expenditures", Quarterly.

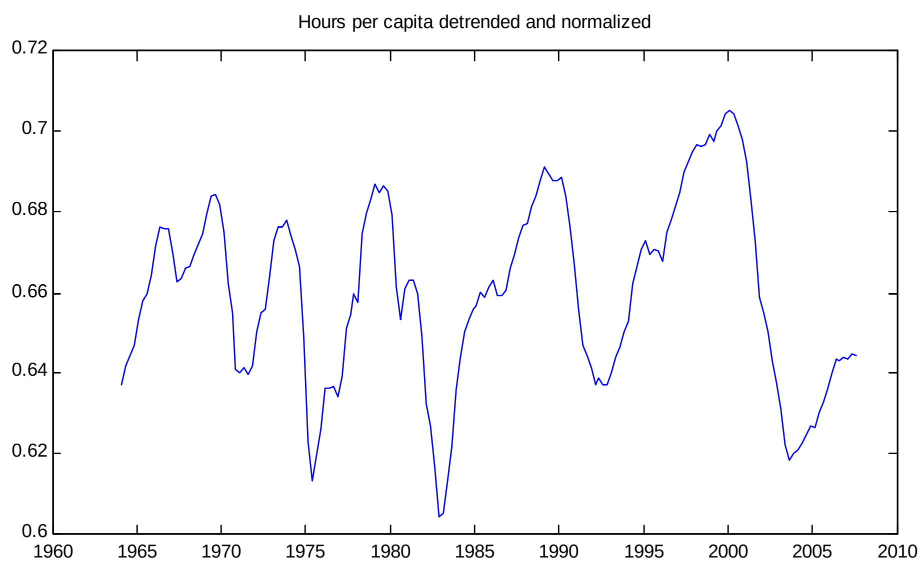
As a measure of Investment I use "Real Gross Private Domestic Investment", Quarterly.

As a measure of Hours I use "Aggregate Weekly Hours Index: Total Private Industries", Monthly.

All data is seasonally adjusted. Monthly data is averaged to make it Quarterly. All the four series are divided by population to obtain per capita values. This corresponds to modelling the economy using a representative household/worker.

Per capita Aggregate Weekly Hours have a linear trend. I divide the per capita hours by the linear trend and normalize the starting value to match the calibration $L = 0.66$. I take logs of GDP, Consumption and Investment, and then take the first difference. All data is for the period 1964:I-2007:III. The next two graphs plot the four data series used for estimation:





Steady-State Calibration

To compute the steady-state one needs to use the following algorithm.

0) set steady-state values for b and L

1) set the shock steady-state values: $a = a \qquad \tau = \tau \qquad d = 0 \qquad z = a^{\frac{1}{1-\alpha}} \tau^{\frac{\alpha}{1-\alpha}}$

2) define constants: $\phi = \left(\frac{\frac{z}{\beta} - \frac{1-\delta_K}{\tau}}{\alpha a} \right)^{-\frac{1}{1-\alpha}} \qquad \eta = a\phi^\alpha - \phi \left(z - \frac{1-\delta_K}{\tau} \right) \qquad \omega = \frac{b}{\theta^\theta (1-\theta)^{1-\theta}}$

3) compute the steady-state value of $B = \omega \frac{1-\beta(1-\delta_L)}{\frac{1}{\eta L}(1-\alpha)a\phi^\alpha - \chi L^\gamma}$

4) compute the steady-state values of the rest of the variables:

$c = \eta L \qquad k = \phi L \qquad U = \delta_L \frac{L}{B} \left(\frac{\theta}{1-\theta} \right)^{1-\theta} \qquad V = \frac{1-\theta}{\theta} U$

The benchmark parametrization I use is borrowed from the papers cited above and includes the values:

$[\alpha = 0.34, \chi = 1.204, b = 0.83, \beta = 0.99, \delta_K = 0.0314, \delta_L = 0.1, \theta = 0.4, L = 0.66, \gamma = 0]$

The trend values are matched to the data means. Notice that:

$d \log Y = \log a z^\alpha \tau^\alpha = \log 1.0053$

$d \log C = \log z = \log 1.0060$

$d \log I = \log z\tau = \log 1.0073$

Therefore, we can compute:

$\tau = 1.0073/1.0060 = 1.\,0013$

$a = 1.0053/1.0073^{0.34} = 1.0028$

The steady-state values are therefore:

1) $\quad z = a^{\frac{1}{1-\alpha}} \tau^{\frac{\alpha}{1-\alpha}} \Big|_{\alpha=0.34, a=1.\,0013, \tau=1.0028} = 1.0034$

2) $\quad \omega = \frac{b}{\theta^\theta (1-\theta)^{1-\theta}} \Big|_{b=0.83, \theta=0.4} = 1.627$

$$\phi = \left(\frac{\frac{z}{\beta} - \frac{1-\delta_K}{\tau}}{\alpha a} \right)^{-\frac{1}{1-\alpha}} \Bigg|_{\alpha=0.34, \beta=0.99, \delta_K=0.0314, a=1.0013, \tau=1.0028, z=1.0034} = 19.681$$

$$\eta = a\phi^\alpha - \phi \left(z - \frac{1-\delta_K}{\tau} \right) \Big|_{\alpha=0.34, \delta_K=0.0314, \theta=0.4, a=1.0013, \tau=1.0028, z=1.0034, \phi=19.681} = 2.0195$$

3) $$B = \omega \frac{1-\beta(1-\delta_L)}{\frac{1}{\eta L}(1-\alpha)a\phi^\alpha - \chi L^\gamma} \Bigg|_{\alpha=0.34, \chi=1.204, \omega=1.627, \beta=0.99, \delta_L=0.1, a=1.0013, L=0.66, \phi=19.681, \eta=2.0195, \gamma=0} = 1.0980$$

4) $\quad c = \eta L|_{L=0.66, \phi=19.681, \eta=2.0195} = 1.\,3329$

$k = \phi L|_{L=0.66, \phi=19.681, \eta=2.0195} = 12.\,989$

$U = \delta_L \frac{L}{B} \left(\frac{\theta}{1-\theta} \right)^{1-\theta} \Big|_{\delta_L=0.1, \theta=0.4, L=0.66, B=1.0980} = 0.04713$

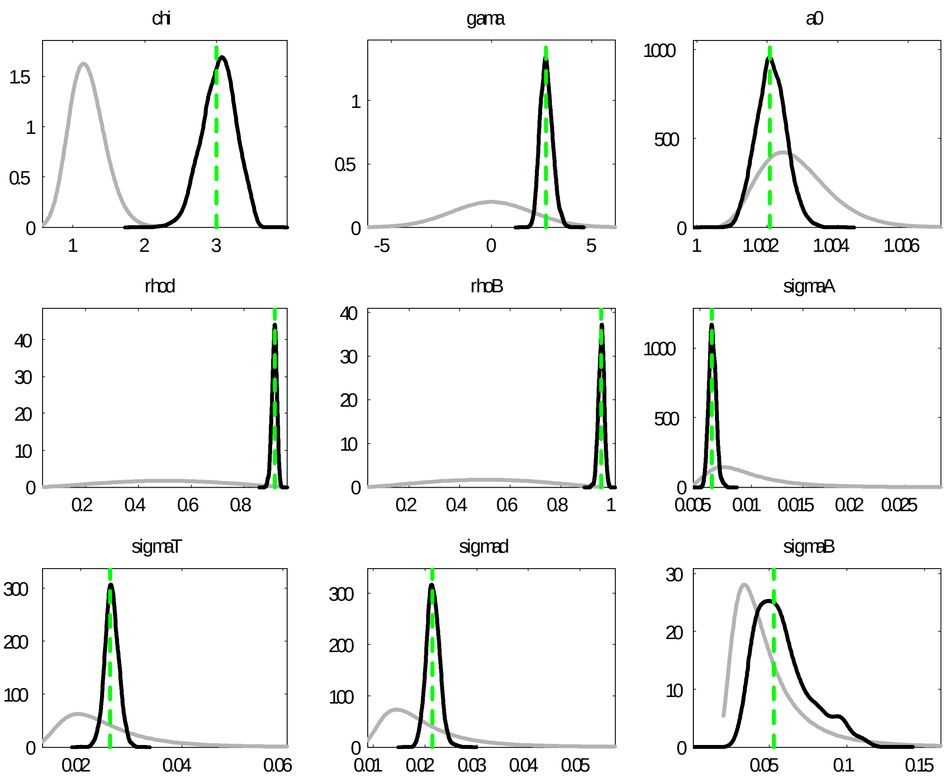
$V = \frac{1-\theta}{\theta} U \Big|_{U=0.04713, \theta=0.4} = 0.0707$

Estimation

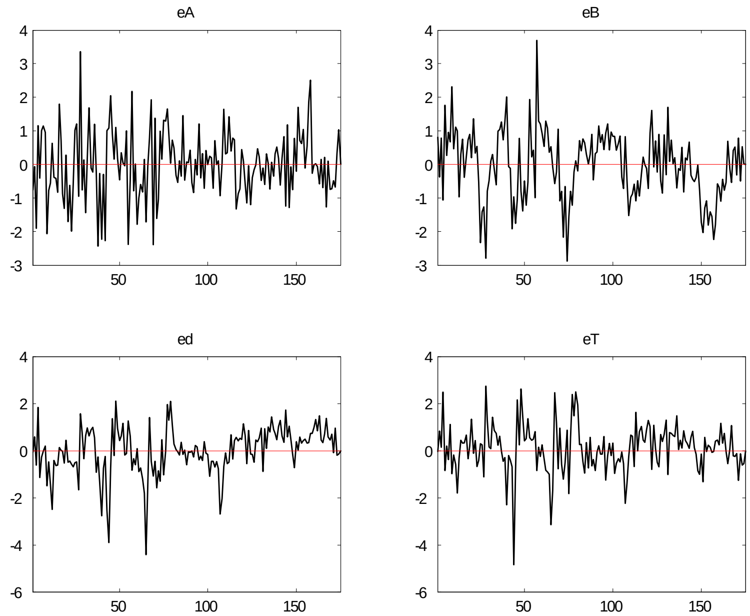
I use Dynare to estimate the parameters of the model with the Metropolis-Hastings algorithm. First I simulate data and estimate the parameters using artificially generated data. This allows me to test for identification. Parameters $[\alpha, \beta, \theta, \delta_K, \delta_L, \omega]$ are not identified. The rest are (at least weakly) identified:$[\chi, \gamma, a, \tau, \rho_d, \rho_B, \sigma_A, \sigma_T, \sigma_d, \sigma_B]$. For convergence purposes I also fix the growth rate of investment-specific technology τ. I perform 5 blocks 5000 iterations each. The acceptation rate is around 34%. Estimation results are summarized in the table:

	Prior distr.	Prior mean	Prior s.d.	Post. mode	s.d.	Post. mean	HPD inf	HPD sup
χ	gamm	1.204	0.2500	2.9954	0.2608	3.0265	2.6404	3.3928
γ	norm	0.000	2.0000	2.7200	0.3081	2.7390	2.2388	3.1992
a	gamm	1.003	0.0010	1.0021	0.0006	1.0021	1.0014	1.0027
ρ_d	beta	0.500	0.2000	0.9174	0.0098	0.9154	0.9004	0.9298
ρ_B	beta	0.500	0.2000	0.9598	0.0111	0.9601	0.9402	0.9762
σ_A	invg	0.010	0.0050	0.0062	0.0003	0.0062	0.0057	0.0068
σ_T	invg	0.025	0.0100	0.0258	0.0014	0.0262	0.0241	0.0285
σ_d	invg	0.020	0.0100	0.0214	0.0014	0.0215	0.0194	0.0235
σ_B	invg	0.050	0.0300	0.0531	0.0164	0.0588	0.0321	0.0874

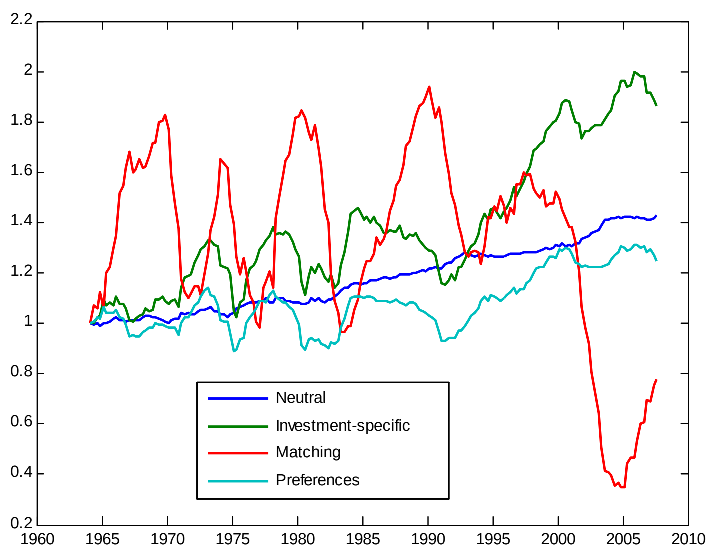
Posterior densities:



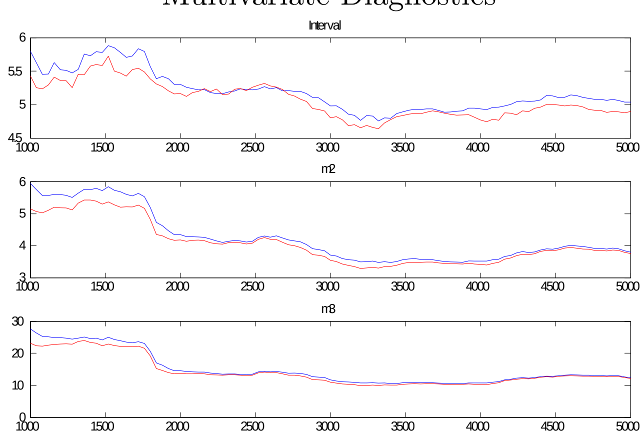
Smoothed Shocks



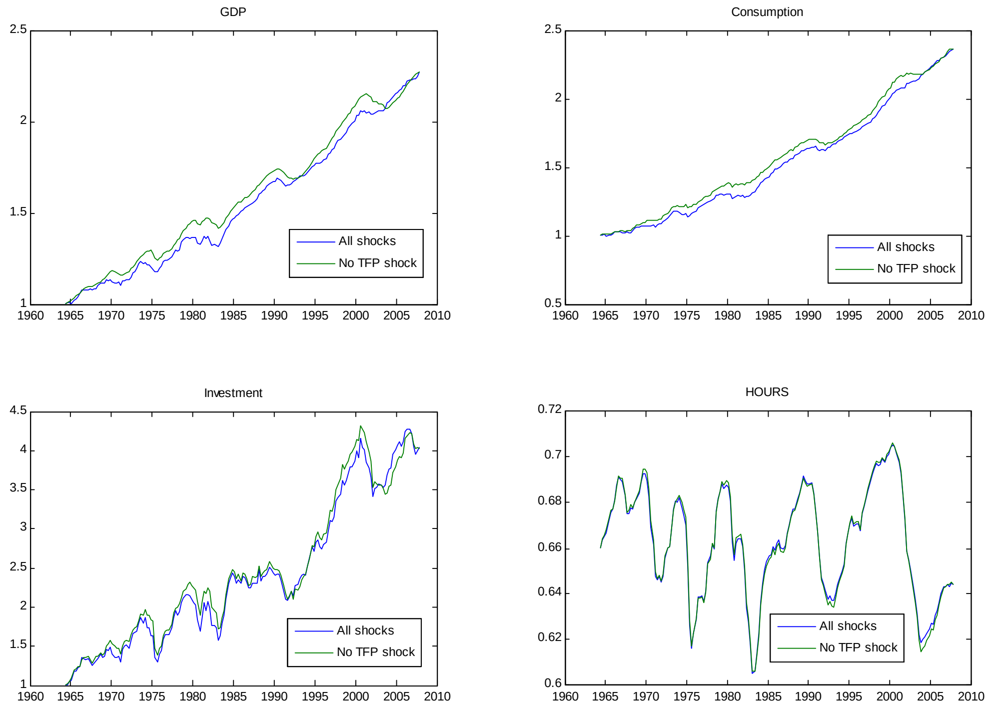
Smoothed shock processes



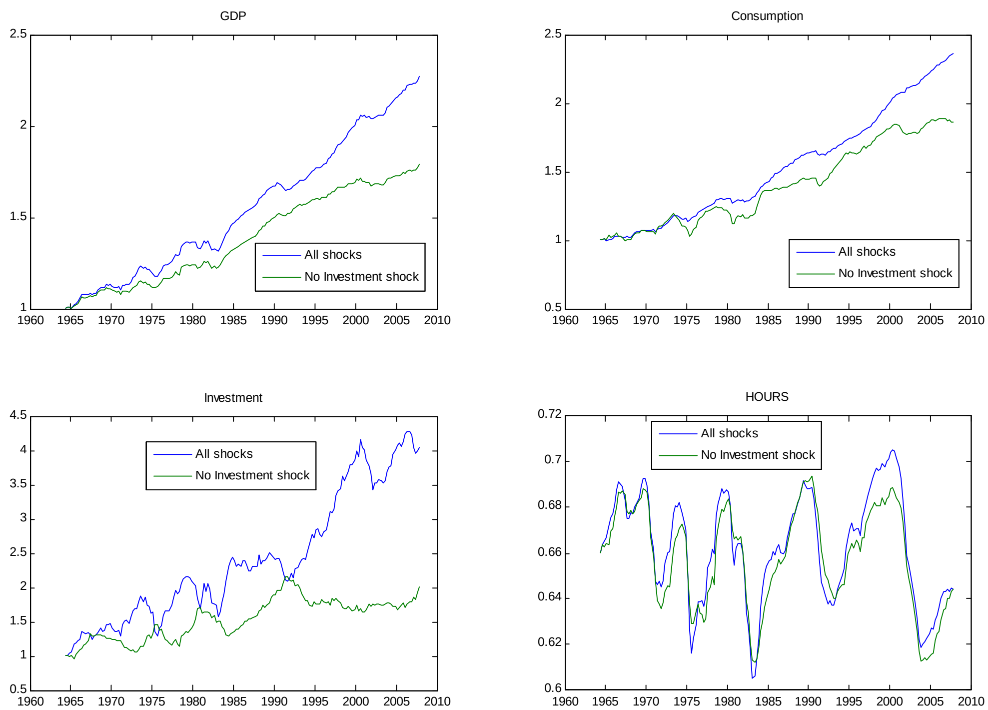
Multivariate Diagnostics



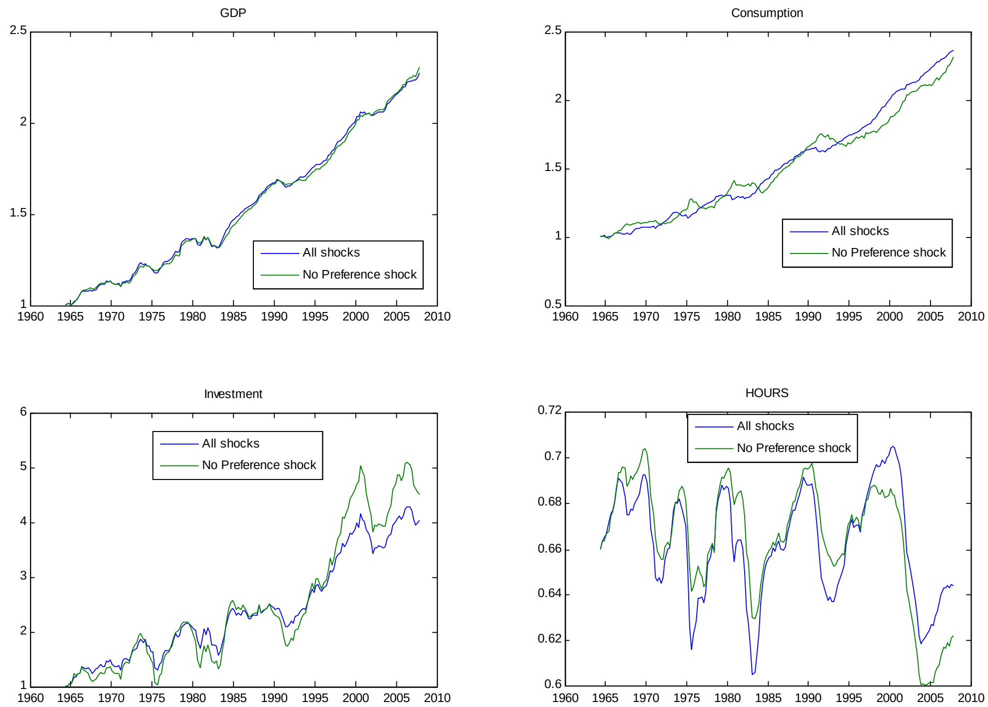
Paths in absense of the shock to TFP



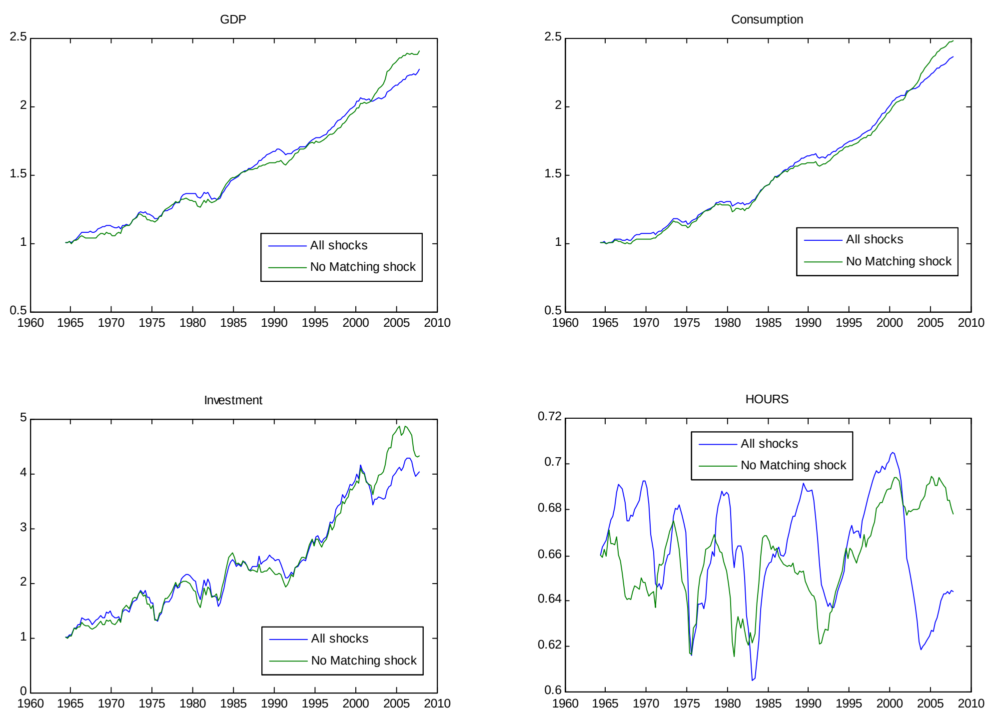
Paths in absense of the shock to investment productivity



Paths in absense of the preference shock



Paths in absense of the matching shock



Discussion

The model fits the data exactly because the number of shocks is exactly equal to the number of observed variables. This exercise is similar in spirit to the business-cycle accounting framework developed by Chari, Kehoe and McGrattan (2007). Unlike that framework I estimate the parameters of the model, that are identified.

The parameters appear to be quite different from what calibration exercises would give. For instance, my estimates of the labor preferences tend to put more weight on disutility of labor. They also indicate, that preferences for labor are significantly nonlinear ($\gamma > 0$ and $\chi > 2$).

Another novelty of this model is that it is the first to use Bayesian techniques to estimate an RBC model with labor accumulation through search and matching. I estimate the variance of shocks to the matching technology to be big (up to 6%).

The aim of the model was to distinguish between the impact of each of the shocks. The above figures provide a graphical characterization of the importance of each of the shocks. I compare the data to the model-generated data in the absense of one of the shocks.

The impacts of the TFP shock and the Preference shock are small and have only a temporary effect on all of the variables. TFP has a negligible effect on hours. Preferences have a negligible effect on output.

The impact of the investment-specific shock is key to the growth and business-cycle properties of the economy. It is the main source of growth over the whole fourty-year period. All the recessions seem to have something to do with the investment-specific shock as well.

The impact of the matching shock is very surprizing. Though it only had a minor effect before 2000, it seems to explain the depth and persistence of the recession of 2001. It also drives the dynamics of hours.

References

References

[1] Roger E. A. Farmer and Andrew Hollenhorst (2006), "Shooting the Auctioneer", NBER Working Paper No. 12584

[2] Jesus Fernandez-Villaverde and Juan Rubio-Ramirez (2006), "Estimating Macroeconomic Models: A Likelihood Approach", CEPR Discussion Paper No. 5513.

[3] V. V. Chari, Patrick J. Kehoe and Ellen R. McGrattan (2007), "Business Cycle Accounting", Econometrica, Vol. 75, No. 3, May 2007, pp. 781-836.

[4] Federal Reserve Bank of St. Louis: Economic Data - FRED (http://research.stlouisfed.org/fred2/).