John Riley 201A Mid-term 24 October 2006

Attempt question 1 and any two other questions. Each will be graded out of 10 points. For the very few who may finish early you can try a 4th question but it will be worth only 5 points. You must indicate which is your "bonus" answer.

1. Choice over time

There are H identical consumers. Consumer h has period t consumption vector $x_t^h = (x_{t1}^h, x_{t2}^h)$ and lifetime utility function $U^h(x_1^h, x_2^h) = u(x_1^h) + u(x_2^h)$ where $u(x_t) = \ln x_{t1} + 2\ln x_{t2}$. The endowment is $\omega = (100, 100, 0, 200)$. Each unit of commodity 1 in period 1 can be either consumed or transformed into 0.5 units of commodity 1 in period 2. Commodity 2 is not storable.

2 (a) Explain briefly why this economy can be analyzed as if there is a single consumer. commodity 1 at t=1 commodity 1 at t=2

1 (b) Write down the relationship between x_{t1} and x_{t2}. x_{11} consumption of

2 (c) Solve for the optimal period 1 consumption of commodity 1 and hence the final allocation.

2 (d) Solve for the WE spot and futures prices. (Use the normalization $p_{11} = 1$.)

2 (e) Solve for the WE future spot prices and interest rate if $p_{21}^{FS} = p_{11} = 1$.

1 (f) If the second commodity could be costlessly stored, would any of it be stored?

2. Pareto Efficient (PE) Allocations

There are two consumers, Alex and Bev. Alex has utility function $U^A = (10 + x_1^A)^{1/2} + (x_2^A)^{1/2}$. Bev has utility function $U^B = (x_1^B)^{1/2} + (x_2^B)^{1/2}$. The aggregate endowment is $\omega = (90, 100)$.

5 (a) Solve for the PE allocations in the interior of an Edgeworth Box and depict these in a neat figure.

3 (b) Indicate the other PE allocations and prove your claim.

1 (c) If Alex has the entire endowment what is the Walrasian Equilibrium (WE) price ratio?

1 (d) If Bev has the entire endowment, what would be the WE price ratio?

3. Production and Cost

A firm has production set $Y = \{(z,q) \mid z_1 z_2 \geq q,\ z \geq 0\}$ where z is the input vector and q is output.

6 (a) Given the input price vector r, solve for the cost function $C(q,r)$.

4 (b) Suppose instead that there are three inputs and that the production set is $Y = \{(z,q) \mid z_1(z_2 + z_3) \geq q,\ z \geq 0\}$. Again solve for the cost function for all possible price vectors $r = (r_1, r_2, r_3)$.

4. Short and Long Run Responses to an input price change

A firm is a monopolist in its output market but a price-taker in its input markets.

4 (a) Show that if the price of input 1 rises, the monopoly will purchase less of this input.

3 (b) Is maximized profit a ~~concave~~ convex function of the input price vector? Prove any claim.

3 (c) Does the Le Chatelier principle apply? That is will the change in demand be smaller in the short-run. Explain with the help of a neat figure.