Econ 201A: Mid-term Fall 2005

Time allowed: 85 minutes

1. Profit maximization

(a) Prove that the profit function $\Pi(p) = \underset{y}{Max}\{p \cdot y \mid y \in Y\}$ is convex. You may assume that

± the production set is closed and bounded so there is a maximum for any $p \geq 0$.

+ (b) Discuss briefly what assumption ensures that, for each price, there is a unique profit maximizing production vector $y(p)$. You may assume that this function is differentiable.

+ (c) Appeal to the Envelope Theorem to get an expression for the first and second partial derivatives with respect to p_j of the profit function.

+ (d) Hence or otherwise show that if $y_j(p)$ is an output, it will increase as p_j increases.

+ (e) What if commodity *j* is an input?

2. Equilibrium and Efficiency

Alex has utility function $U^A(x^A) = (x_1^A)^{1/2} + (\alpha + x_2^A)^{1/2}$, $\alpha > 0$. Bev has utility function $U^B(x^B) = (\beta + x_1^B)^{1/2} + (x_2^B)^{1/2}$, $\beta > 0$. The aggregate endowment is ω.

(a) Show that at any PE allocation in the interior of the Edgeworth Box the supporting Walrasian

+ Equilibrium price ratio is the same. What is this price ratio?

+ (b) What is the range of possible WE price ratios in this economy?

(c) Is it the case that, for each possible endowment, there is a unique WE price ratio? Explain carefully.

3. Supply and demand

Owners of firms live off their dividends and supply no labor. Workers do not own shares. There is a single firm that produces corn according to the production function $Q = L^{1/2}$. The price of output is *p* and the wage rate is *w*.

+ (a) Solve for the firm's maximized profit and labor demand.

+ (b) Each worker has a utility function of the form $U = C^2 H = C^2(24 - L)$, where C is

+ consumption, *H* is leisure and *L* is hours worked per day. Solve for the labor supply function.

+(c) Hence solve for the wage-price ratio that clears the labor market.

+(d) Will this also clear the market for corn?