4. Suppose that the regression model is $y_i = \alpha + \beta x_i + \varepsilon_i$, where the disturbances $\varepsilon_i$ have $f(\varepsilon_i) = (1/\lambda)\exp(-\lambda\varepsilon_i)$, $\varepsilon_i \geq 0$. This model is rather peculiar in that all the disturbances are assumed to be positive. Note that the disturbances have $E[\varepsilon_i \mid x_i] = \lambda$ and $\text{Var}[\varepsilon_i \mid x_i] = \lambda^2$. Show that the least squares slope is unbiased but that the intercept is biased.
5. Prove that the least squares intercept estimator in the classical regression model is the minimum variance linear unbiased estimator.
6. As a profit maximizing monopolist, you face the demand curve $Q = \alpha + \beta P + \varepsilon$. In the past, you have set the following prices and sold the accompanying quantities:

| **Q** | 3 | 3 | 7 | 6 | 10 | 15 | 16 | 13 | 9 | 15 | 9 | 15 | 12 | 18 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **P** | 18 | 16 | 17 | 12 | 15 | 15 | 4 | 13 | 11 | 6 | 8 | 10 | 7 | 7 | 7 |

Suppose that your marginal cost is 10. Based on the least squares regression, compute a 95 percent confidence interval for the expected value of the profit maximizing output.
7. The following sample moments for $x = [1, x_1, x_2, x_3]$ were computed from 100 observations produced using a random number generator:

$$\mathbf{X}'\mathbf{X} = \begin{bmatrix} 100 & 123 & 96 & 109 \\ 123 & 252 & 125 & 189 \\ 96 & 125 & 167 & 146 \\ 109 & 189 & 146 & 168 \end{bmatrix}, \quad \mathbf{X}'\mathbf{y} = \begin{bmatrix} 460 \\ 810 \\ 615 \\ 712 \end{bmatrix}, \quad \mathbf{y}'\mathbf{y} = 3924.$$

The true model underlying these data is $y = x_1 + x_2 + x_3 + \varepsilon$.
a. Compute the simple correlations among the regressors.
b. Compute the ordinary least squares coefficients in the regression of $y$ on a constant $x_1$, $x_2$, and $x_3$.
c. Compute the ordinary least squares coefficients in the regression of $y$ on a constant $x_1$ and $x_2$, on a constant $x_1$ and $x_3$, and on a constant $x_2$ and $x_3$.
d. Compute the variance inflation factor associated with each variable.
e. The regressors are obviously collinear. Which is the problem variable?
8. Consider the multiple regression of $\mathbf{y}$ on $K$ variables $\mathbf{X}$ and an additional variable $\mathbf{z}$. Prove that under the assumptions A1 through A6 of the classical regression model, the true variance of the least squares estimator of the slopes on $\mathbf{X}$ is larger when $\mathbf{z}$ is included in the regression than when it is not. Does the same hold for the sample estimate of this covariance matrix? Why or why not? Assume that $\mathbf{X}$ and $\mathbf{z}$ are nonstochastic and that the coefficient on $\mathbf{z}$ is nonzero.
9. For the classical normal regression model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ with no constant term and $K$ regressors, assuming that the true value of $\boldsymbol{\beta}$ is zero, what is the exact expected value of $F[K, n-K] = (R^2/K)/[(1-R^2)/(n-K)]$?
10. Prove that $E[\mathbf{b}'\mathbf{b}] = \boldsymbol{\beta}'\boldsymbol{\beta} + \sigma^2 \sum_{k=1}^{K}(1/\lambda_k)$ where $\mathbf{b}$ is the ordinary least squares estimator and $\lambda_k$ is a characteristic root of $\mathbf{X}'\mathbf{X}$.
11. Data on U.S. gasoline consumption for the years 1960 to 1995 are given in Table F2.2.
a. Compute the multiple regression of per capita consumption of gasoline, $G/pop$, on all the other explanatory variables, including the time trend, and report all results. Do the signs of the estimates agree with your expectations?