Creative Destruction under Rational Inattention

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Abstract

We show new theory and evidence on the link between business failures, markups and business cycle asymmetry in the U.S. economy. We study a model where costly information-processing constraints affect exit decisions of heterogeneous firms in the presence of an aggregate demand externality. We show that such a model is capable of explaining both the novel and the classical empirical evidence on output growth asymmetry, the asymmetry between entry and exit rates, as well as counter-cyclical variations in profit margins.

JEL: D81, E32, D21, D40, C44, C63.

Keywords:Information, Expectations, Business Formation, Markups, Rational Inattention.

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1 Introduction

A recurring feature of business activity in the U.S. economy is the asymmetry between expansions and contractions. The economy tends to alternate between long periods of slow expansion and short periods of sharp contraction.[1] In periods of expansion the economy experiences a net increase in the number of firms which create new products and jobs. These periods of slow expansion are occasionally interrupted by contraction episodes, when large numbers of firms simultaneously layoff workers and go out of business.

Firms' decisions to enter and exit the market affect business cycle variations through their impact on the overall number of firms in the market and, as a result, the degree of competition. For instance, Jaimovich and Floetotto (2008) document that the number of firms is strongly procyclical, and entry and exit decisions account for a substantial fraction of jobs created and destroyed in the U.S. economy. Thus, these decisions are an important source of business cycle fluctuations in the U.S. economy. While the majority of the theoretical and empirical literature has focused on the interaction between entry decisions and cyclicality of firms' markups at business cycle frequencies,[2] available data for the U.S. economy indicates that exit rates are highly correlated with the business cycle and highly positively skewed. Yet, a link between asymmetry of business cycles, markups and firms' exit decisions is to be extablished.

This paper argues that exit decisions play a pivotal role in explaining asymmetric behavior of profit margins and aggregate activity in the U.S. The paper makes two contributions to the existing literature. First, we present novel empirical evidence on the relationship between asymmetric variations in the number of market participants and asymmetric behavior of markups. We document that variations in exit decisions of firms are the key source of variations in the number of market participants. Thus, exit decisions are shown to be a major source of asymmetry in the business cycle.

Second, we propose a theoretical framework based on rational inattention theory of Sims (2006) and show that its predictions are consistent with both classical and novel empirical evidence. We formally model endogenous exit

[1] According to the NBER's business cycle dating committee, since 1900 the average length of expansions (11 quarters) has been three times longer that the average length of contractions (3.6 quarters).

[2] For instance, Bilbiie Ghironi and Melitz (2008) model variations in the number of firms as coming from the entry margin alone.

decisions of heterogeneous firms in the presence of information processing constraints and an aggregate demand externality. In our setup, firms can choose to pay attention to a joint signal on aggregate conditions and on firm specific demand. We focus on (1) which part of this information firms choose to pay relatively more attention to, (2) how this new information affects their perception of market conditions, and (3) how it affects their exit decisions.

Four main predictions emerge from the model. First, paying attention to information on both aggregate and idiosyncratic conditions becomes relatively more costly as profit margins fall. This makes monitoring aggregate conditions harder and leads to aggregate reduction in attention as the economy expands.

Second, we find that firms increase the share of attention allocated to monitoring aggregate conditions when increased competition lowers average profit margins. This is because firms choose an optimal signal in which the aggregate component is correlated with the idiosyncratic component to the extent that information processing constraints allow it. Hence, after observing profits, precise information about the aggregate component allows firms to identify the idiosyncratic component. Firms choose to pay most of their attention to aggregate conditions because they are more persistent and, as a result, less costly to track. Information about aggregate conditions gives firms information about choices of its competitors.

Third, because firms base their perceptions on similar signals about aggregate conditions, these signals work as a coordination device: conditions which trigger exit tend to occur simultaneously to many firms. These conditions take the form of a probabilistic perception of a signal about consumer's taste for their product. The probability of perceiving the product as out of fashion and demand as low increases sharply as aggregate conditions vary.

Fourth, the model economy predicts counter-cyclical positively skewed variations in profit margins. A test of these predictions using data on price-cost markups recently constructed by Nekarda and Ramey (2010) overwhelmingly supports the positive growth rate asymmetry of markups. We argue that this last finding of highly positively skewed markups can shed new light on the debate over the cyclical properties of markups in the U.S. economy. The rationale behind this claim is that our measure is much more robust than commonly used correlations because it uses the distribution of growth rates as opposite to relying on the way the series for markups and GDP are de-trended, as common in the literature.

From an empirical perspective, Campbell and Hopenhayn (2005) provide strong empirical evidence on the interaction between markups and market tightness as expressed by the number of competitors. Moreover, Devereux et al. (1996) show that exit rates are strongly counter-cyclical. Using annual data by industry for the U.S. economy between 1958 and 1992, Jaivomich and Floetotto (2008) provide evidence on the relationship between business failures and real GDP.

From a theoretical perspective, a basic framework for the analysis of variations in net business formation has been proposed by Jaimovich (2007). He shows that variations in net business formation over the business cycle can lead to counter-cyclical variations in profit margins. A consequence of the interaction between net business formation and market power is the possibility of self-fulfilling expectations-driven fluctuations. Jaivomich and Floetotto (2007) use a dynamic general equilibrium model where endogenous procyclical entry of firms lead to countercyclical markups.

Building upon one sector of Jaimovich's economy, this paper explores the link between endogenous firm exit and market power. We introduce taste shocks to firm products which are the source of heterogeneity across firms and the only exogenous source of uncertainty in the model. Instead of assuming an aggregate zero-profit condition which immediately pins down the number of firms, we study the dynamic endogenous exit decisions of firms.[3]

To understand the effects of information on expectation formation and on the behavior of our model economy we introduce information processing constraints. In this environment, a natural framework to analyze information processing and expectation formation is the rational inattention framework of Sims (2006). We adopt the view that while all information is available, the fact that processing information is costly leads to endogenous variations in the way firms form expectations.

Prior to their exit decisions we let firms endogenously choose how much information to process to form expectations about future market conditions, while fully taking into account the consequences of this choice. This endogenous choice of expectations together with individual firms' prior beliefs and the shape of their profit function pin down one of the multiple possible dynamic equilibria that will prevail. We then study the effects of changes in the cost of processing information and other parameters on the dynamic behavior of our model economy.

As a first step we show that under full information (zero cost of processing

[3] For tractability we assume that the number of new entrants is fixed exogenously.

information) firms are able to identify the best time to exit, which leads to symmetric aggregate fluctuations. The full-information economy does not exhibit any asymmetries in output growth or large difference between the behavior of entry and exit.

We then turn to the case of costly information processing. We find that when processing information is costly, firms are unable to correctly identify the optimal time to exit. Because processing information takes time, firms tend to stay in the market for too long, until they realize that aggregate conditions are bad enough, and then exit in a flock. In the aggregate, this behavior leads to asymmetric cyclical patterns in the constrained economy. Such patters are a consequence of the information constraint only.

Information plays a dual role in this economy. On the one hand, the cost of processing information delays the exit decision of a firm by making it harder to identify the optimal time to go. On the other hand, information about aggregate conditions plays a coordinating role by making individual perceptions and decisions very similar and, thus, generating large numbers of simultaneous exits. The way information aggregation works in our model is a special case of the pure information externality model of Chamley and Gale (1994) and its extension by Murto and Valimaki (2011). In these papers, aggregation of information dispersed across a large population of firms results in swift rushes of firms' exits. Learning about the optimal stopping time of others is the key mechanism through which information is dispersed across the population. Much like Murto and Valimaki (2011), our model predicts exit waves due to information aggregation. Different from this paper, in our framework the mechanism through which learning occurs comes from endogenous firms' choices of information structure.

Endogenous information structure arising from rational inattention models commands a tall computational burden. For tractability, we limit the analysis to a finite number of firms.

While at first glance our model looks more suited for the analysis of firm dynamics in a particular industry along the lines of Ericson and Pakes (1995), we argue that some of its key implications carry over to the whole economy. We show that due to granularity of the U.S. economy, as documented by Gabaix (2011), a substantial increase in the number of firms in the model, while not feasible computationally, would lead to only mild changes in the behavior of normalized moments such as correlations and skewness of growth rates. Therefore, the key predictions of the model for skewness of growth rates of output, markups, entry and exit rates can be meaningfully compared to aggregate data.

Numerical results show that our information constrained economy can reproduce qualitatively the behavior of the U.S. economy, alternating from periods of slow expansions and accumulation of businesses to sharp downward adjustments associated with business shredding. First, we show that the model economy reproduces the well documented negative growth rate asymmetry present in the U.S. economy. Second, our model economy can account for the relationship between the highly positively skewed countercyclical firm exit rates and the positive growth asymmetry of markups in the U.S. economy.[4]

This paper contributes to three strands of literature. First, we contribute to the literature aimed at explaining business cycle asymmetries. Like Shleifer (1986), Zeira (1994), Matsuyama (1999), Francois and Lloyd-Ellis (2003) our business cycle mechanism relies on variations in monopoly power in the presence of uncertainty about current or future profitability. Like Jovanovic (2006), Zeira (1994), and Van Nieuwerburgh and Veldkamp (2006), our explanation of the asymmetries rests on variations in the amount of information processed by agents in the economy. Like in Chamley and Gale (1994) and Murto and Valimaki (2011), an information externality results in nonlinear information aggregation which leads to exit waves. Unlike most of these studies, the key element of our model is firms' endogenous choice of information structure.

Second, we contribute to the literature which studies effects information processing and belief formation have on aggregate fluctuations. Mankiw and Reis (2002) study sticky information and its effect on price variations. Lorenzoni (2009) analyzes the role of heterogeneity in consumer expectations as the driving force behind aggregate demand. There is also a large related literature on coordination in the presence of externalities[5] which shows that properties of equilibria in coordination games are affected by the availability and precision of information and, more generally, by restrictions on information acquisition placed by the environment.

Our paper differs methodologically from most of these papers. There is an important difference between a 'beauty contest' coordination game and the business formation game we study in this paper. While in a 'beauty

[4]The model by construction replicates flat and almost a-cyclical firm entry rates.

[5]See Morris and Shin (2002), Angeletos and Pavan (2007), Hellwig and Veldkamp (2009), Myatt and Wallace (forthcoming).

contest' game agents learn about an aggregate exogenous state of the economy, in our framework exogenous agent-specific processes through their effect on agent's exit decisions determine the aggregate state of the economy. A natural framework to analyze information processing and expectation formation in this situation is the rational inattention framework proposed by Sims (2006).

Unlike Mackowiak and Wiederholt (2009), who apply this framework to analyze price stickiness, we do not rely on Gaussian distributions. Instead, the optimal joint distribution of attention is fully endogenous. This approach has already proven useful in Tutino's (2011) analysis of asymmetries in consumption-savings decisions. One of the contributions of this paper is to provide a new way of thinking about general equilibrium under rational inattention and solving for this dynamic equilibrium numerically.

Finally, our mechanism generates counter-cyclical variations in profit margins, which are related to the idea of competitive wars first outlined by Rotemberg and Saloner (1986). Bilbiie, Ghironi and Melitz (2008) relate variations in markups to endogenous variations in entry and product variety, while Edmond and Veldkamp (2009) analyze the interplay of variations in the degree of heterogeneity and counter-cyclical markups. Using data on markup variations constructed by Nekarda and Ramey (2010), we find strong empirical support for these counter-cyclical theories of markups using a measure of markup growth asymmetry, which does not rely on the way the data is de-trended. However, unlike the mechanisms described earlier, most of the business cycle adjustments in our model occurs on the exit (rather than entry) margin. This is consistent with evidence on the behavior of firms entry and exit rates in the U.S. economy, which we document in this paper.

The paper is organized as follows. Section 2 presents novel empirical regularities in the U.S. economy regarding markups and exit rates. In Section 3, we describe the primitives of the model, examine in detail the information structure, and the problem of the firm. We explain in detail how a capacity constraint on information processing modifies the problem of the firm. In Section 4, we calibrate the model and describe the results of simulations of different versions of the model. In Section 5, we explore the mechanism of information processing and coordination under rational inattention. Section 6 discusses the sensitivity of our results to various changes in parameters and tests the main predictions of the model using U.S. data. We conclude by discussing potential policy implications of our business cycle mechanism.

2 Empirical Evidence

In this section we describe business cycle properties of markups, exit and entry rates of firms in the U.S. economy and document some new stylized facts about their asymmetric properties. To describe the first set of facts we utilize quarterly data on markups for the U.S. economy from 1948:1 to 2010:4 constructed by Nekarda and Ramey (2010).[6]

Fact 1. *Markups lag the business cycle. Lagged markups are countercyclical.*

In Table 1 we present tests of Granger causality for real GDP and markups, as well as their growth rates. Table 1 shows that we can reject the hypothesis that variations in real GDP do not Granger cause variations in markups and that variations in growth rates of GDP do not Granger cause variations in growth rates of markups. The opposite statements cannot be rejected. Figure 1 reports correlations of leads and lags of de-trended levels and growth rates of real GDP and markups. It shows that both lags of markups and lags of their growth rates are significantly negatively correlated with real GDP.

Fact 2. *Markups show a strong positive growth rate asymmetry, the opposite of real GDP.*

Using the same data, we report skewness of GDP growth and markup growth in Table 1. While the fact that real GDP growth is negatively skewed has been widely documented in the literature, the strong positive skewness of markups is a new finding.

Fact 1 indicates that monopolistic competition plays an important role in business cycles. This is consistent with a large body of theoretical work,[7] which stresses the importance of markup variations for understanding business cycles. Fact 2 indicates that markup variations are also highly asymmetric. Markups rise steeply following recessions and fall gradually in expansions. This new finding suggests that understanding monopolistic competiton is crucial to understanding business cycle asymmetries.

The finding of high positive skewness of changes in markups sheds new light on the controversial behavior of markups over the business cycle. While

[6] Simply using the ratio of revenues in GDP to employee compensation in GDP leads to similar results.

[7] See Bilbiie Ghironi and Melitz (2008), Jaimovich (2007) and Jaimovich and Floetotto (2008).

Rotemberg and Woodford (1999) find markups to be countercyclical, Nekarda and Ramey (2010) recently argued that markups are virtually a-cyclical. These findings rely on computing correlations of de-trended time series, which makes them particularly sensitive to the statistical model for the mean used to de-trend markups and GDP. The facts we document in this paper are robust to this concern because they can be established purely based on properties of first-differenced data, which is independent of the model for the mean or a de-trending procedure.

We thus provide strong empirical support to a counter-cyclical theory of markups. This theory implies that markups rise sharply in the aftermath of a recession due to a decline in the number of competitors, and then fall gradually in a boom as new businesses populate the economy. This interpretation is consistent with the pattern of cross-correlations of growth rates of GDP and markups, and with correlations of de-trended levels depicted in depicted in Figure 1.

Variations in the number of firms are a major contributor to variations in markups through their effect on market tightness. Procyclicality of the number of firms is well documented.[8] However, in modeling the relationship between the number of firms and markups the literature has either abstracted from explicitly modeling entry and exit, or focused only on entry of new firms.[9] To understand asymmetry of markups it is necessary to understand the cyclical and asymmetric properties of both exit and entry of firms.

To describe the properties of entry and exit rates we use quarterly data on the number of opening and closing establishments in the U.S. economy reported by the Business Employment Dynamics survey. To check the robustness of these properties we build on the findings of Jaimovich and Floetotto (2008) who show that closing and opening establishments account for a large fraction of cyclical variations in job destruction and job creation rates. As we document in Table 2 and illustrate in Figure 2, the correlation between exit and job destruction rates is 0.71, and the correlation between entry and job creation rates is 0.65. This suggests that job destruction and creation rates are informative proxies for the numbers of exiting and entering firms in the U.S. economy. Table 2 summarizes the next three facts.

[8] See, inter alias Chatterjee and Cooper (1993), Devereux et. al. (1996) and Jaimovich and Floetotto (2008).

[9] See, inter alias, Bilbiie Ghironi and Melitz (2008), Jaimovich (2007), and the subsequent literature.

Number of lags	1	2	3	4	5	6
GDP does not Granger cause Markup						
F-statistic	4.6**	7.5***	6.5***	7.3***	9.8***	7.6***
Markup does not Granger cause GDP						
F-statistic	7.2**	0.9	2.3*	1.7	1.9	1.4
d(GDP) does not Granger cause d(Markup)						
F-statistic	10.8***	8.6***	8.7***	11.4***	8.2***	6.2***
d(Markup) does not Granger cause d(GDP)						
F-statistic	0.2	2.5*	1.4	1.4	0.9	2.1*
Skewness of GDP growth					−0.18	
Skewness of Markup growth					+0.68	
Correlation of d(GDP) with lag of d(Markup)					−0.21***	

TABLE 1. **Cyclicality of Markups**

Frequency: Quarterly 1948:1-2010:4. Observations: 251. *, ** and *** show significance at 10%, 5% and 1%. Sources: NIPA; Nekarda and Ramey (2010).

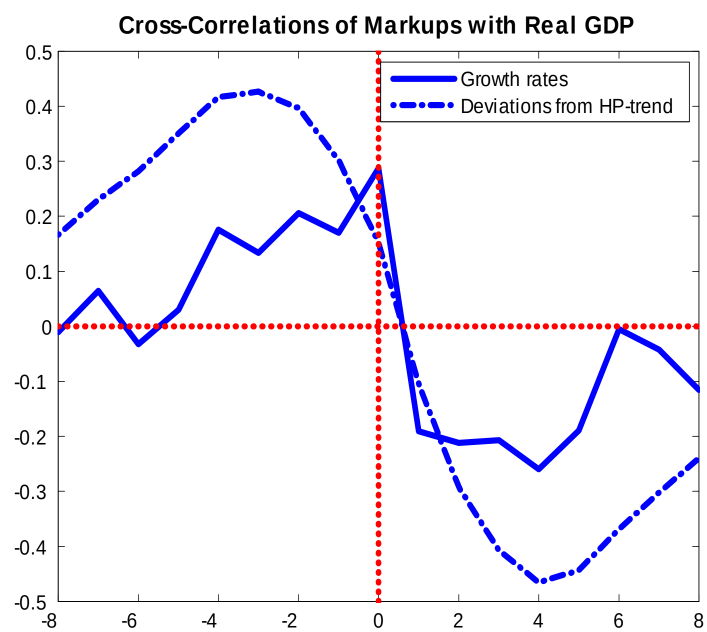

FIGURE 1. **Cyclical Properties of Markups.**

Sources: NIPA, Nekarda and Ramey (2010).

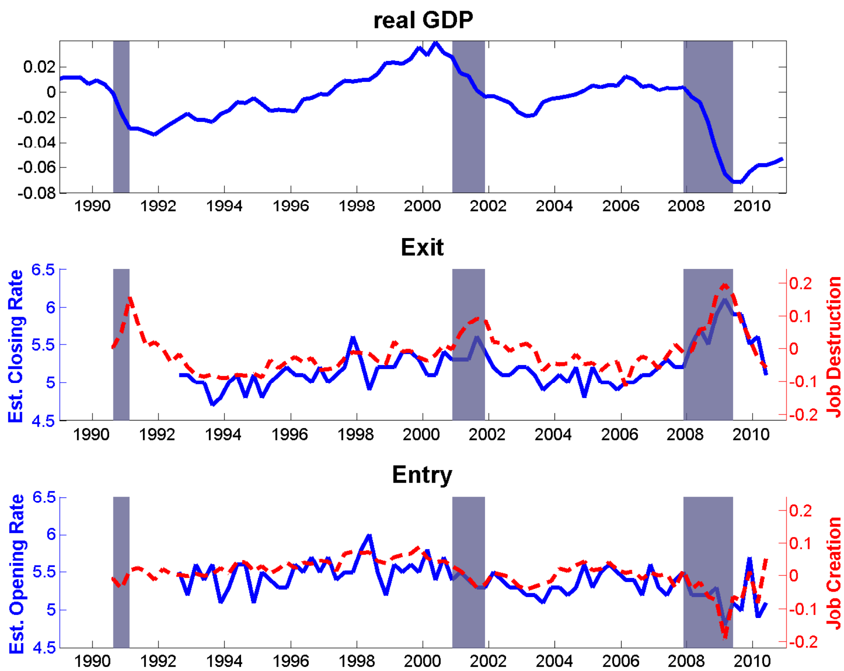

FIGURE 2. **Real GDP, Firm Entry and Exit rates**

Sources: NIPA; Nekarda and Ramey(2010); Business Employment Dynamics.

Fact 3. *Firm exit is at least 30% more volatile than firm entry.*

The average volatility of firm exit rates in the U.S. economy in the last twenty years has been 5.3%, about 30% higher than the volatility of firm entry rates at 4.0%. One consequence of this is that the volatility of job destruction rates has been more than 50% higher that the volatility of job creation rates.

Fact 4. *Firm exit is strongly countercyclical and highly positively skewed.*

Fact 5. *Firm entry is procyclical and symmetric.*

The correlation of firm exit rate with real GDP growth in the U.S. economy in the last twenty years has been as high as -0.49, with skewness of the exit rate at 1.12. The correlation of firm entry rate a mild 0.23 and with skewness at -0.09. Job destruction and creation rates show similar patterns, with the behavior of job creation slightly more cyclical than entry. Table 2 also strongly supports our earlier fingings that lagged markups are countercyclical and highly asymmetric, positively related to exit, and negatively related to entry.

We provide two robustness checks of our results. First, in order to extend the analysis to a longer time period, we use job destruction and creation rates in manufacturing since 1948 from Davis et.al. (2006) as a proxy for exit and entry rates in manufacturing. We combine these with data on markups in manufacturing from Nekarda and Ramey (2010) and with the index of industrial production as a proxy for manufacturing output. Table 3 shows that this data is consistent with our facts. Lagged markups are countercyclical and exhibit a positive growth asymmetry. Job destruction is much more volatile than job creation. Job destruction is countercyclical and highly positively skewed, while job creation is procyclical and symmetric.

Second, we use annual data on the number of business failures and new incorporations in the U.S. economy from the Statistical Abstract of the United States to construct measures of exit and entry for a longer time period. To eliminate trends in these series we use the standard hp-filter. Table 4 shows that business failures are countercyclical and highly positively skewed at business cycle frequencies, while new incorporations are procyclical and symmetric.

Many of the facts that we document in this paper characterize features of the data largely overlooked by much of the theoretical and empirical business cycle literatures. Facts 2 and 4 suggest that the asymmetry of business cycles goes hand in hand with asymmetric behavior of markups and exit rates. Facts 3, 4 and 5 suggest that the exit margin is far more important than the entry margin for understanding the dynamics of the number of firms and the behavior of markups in the U.S. economy. These facts indicate that it is hard to explain both asymmetry and cyclicality of business cycles without modeling explicitly exit decisions of firms. In the next section we construct a relatively parsimonious model which is able to qualitatively explain these facts.

	d(GDP)	d(Mrk(+3))	Exit	Entry	Job Des	Job Cre
Volatility			5.3%	4.0%	5.1%	3.1%
Skewness	-1.07	1.05	1.12	-0.09	1.06	-0.70
Cross-corellations						
d(Markup(+3))	$-.24^{**}$					
Exit rate	$-.49^{***}$	$+.19^{*}$				
Entry rate	$+.23^{**}$	$-.21^{**}$	$-.42^{***}$			
Job Destruction	$-.51^{***}$	$+.35^{***}$	$+.71^{***}$	$-.13$		
Job Creation	$+.49^{***}$	$-.27^{**}$	$-.26^{**}$	$+.65^{***}$	$-.35^{***}$	

Table 2. **Markups, Exit and Entry rates in the U.S.**

Frequency: Quarterly 1992:3-2010:4. Observations: 71. *, ** and *** show significance at 10%, 5% and 1%.

Sources: NIPA; Nekarda and Ramey (2010); Business Employment Dynamics.

	d(Output)	d(Markup(+3))	Job Destruction	Job Creation
Volatility			16.0%	9.2%
Skewness	-0.20	$+0.39$	$+1.11$	$+0.01$
Cross-corellations				
d(Markup(+3))	-0.18^{***}			
Job Destruction	-0.73^{***}	$+0.28^{***}$		
Job Creation	$+0.75^{***}$	-0.16^{***}	-0.59^{***}	

Table 3. **Markups, Exit and Entry rates in U.S. Manufacturing.**

Frequency: Quarterly 1948:1-2010:4. Observations: 251. *, ** and *** show significance at 10%, 5% and 1%.

Source: Board of Governors; Nekarda and Ramey (2010); Davis, Faberman and Haltiwanger (2006).

	d(Output)	d(Markup(+1))	Bus. Fail.	New Inc.
Skewness	-0.38	$+0.16$	$+0.94$	$+0.03$
Cross-corellations				
d(Markup(+1))	-0.56^{***}			
Business Failures	-0.21^{**}	$+0.23^{**}$		
New Incorporations	$+0.25^{**}$	-0.22^{**}	-0.02	

Table 4. **Markups, Business Failures and New Incorporations.**

Frequency: Annual 1948-2010. Observations: 61. *, ** and *** show significance at 10%, 5% and 1%.

Sources: NIPA; Nekarda and Ramey (2010); Dunn&Bradstreet.

3 Model

In this section we construct the simplest possible economy where variations in the number of firms in the market induce an aggregate externality. The key mechanism at play is that changes in the number of firms affect the existing market through their impact on the degree of competition. Hence, this variation generates a negative demand externality for the incumbent firms. Our model economy could be thought of as a single sector version of the economy described in Jaimovich (2007).

The main difference comes from our focus on modeling separately the entry and exit decisions of firms. We replace the assumption that the number of firms is a jump variable determined by a zero profit condition with a setup where individual firms can choose whether to exit the economy. To simplify things we disregard capital as a factor of production. In our economy the stock of available production capacity is a counterpart of capital. We show in the appendix that this setup is equivalent to a setup with imperfect substitution between goods in the presence of fixed costs.

Thus, apart from the determination of the number of firms, our economy is an exact counterpart of the economy analyzed by Jaimovich for the case when capital share approaches 0, elasticity of substitution equals 1, the elasticity of labor supply equals 0, and there is a single representative sector in the economy. This case satisfies the sufficient condition for existence and uniqueness of a steady-state and the necessary condition for multiple equilibria.

3.1 Primitives

Time is discrete and continues forever. In each time period, the model economy consists of a representative household and K_t firms. We denote each firm with the subscript i, where $i = 1, ..., K$. Firm profit, denoted by π_i, is derived from producing and selling differentiated products q_{it} at price p_{it} net of the wage bill, w_t. Firms' profit function amounts to:

$$\pi_{it} = p_{it}q_{it} - w_t l_{it} \tag{1}$$

Firms use identical production functions, which are linear in labor inputs, l_{it}, and have a capacity constraint:

$$q_{it} = Al_{it} \leq A \tag{2}$$

where A is total production capacity. Within each period firms either operate at full capacity, or do not operate.

The representative household trades off leisure for consumption, maximizing a standard utility function:

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma} - 1}{1-\gamma} - E_t \right), \tag{3}$$

with respect to the supply of labor, E_t, and a consumption aggregator, C_t. In (3), γ denotes the coefficient of relative risk aversion. The consumption aggregator weights differentiated products, q_{it}, by their tastes, v_{it}:

$$C_t = \sum_{i=1}^{K_t} v_{it} q_{it}. \tag{4}$$

The household owns all the firms in the economy. It spends wage and profit income on contemporaneous consumption, maximizing utility subject to a budget constraint:

$$\sum_{i=1}^{K_t} p_{it} q_{it} = w_t E_t + \sum_{i=1}^{K_t} \pi_{it}. \tag{5}$$

Maximization yields the following first-order condition, which determines the demand curve for each good indexed by i.

$$p_{it} = w_t C_t^{-\gamma} v_{it}, \tag{6}$$

which is driven by variations in idiosyncratic tastes v_{it}. Variations in tastes are the only source of uncertainty in the economy.

We define a consumption price index as follows:

$$P_t = \frac{1}{C_t} \sum_{i=1}^{K_t} p_{it} q_{it} = w_t C_t^{-\gamma} \tag{7}$$

Let the wage, w_t, be the numeraire. The expression for firms' profits then simplifies to:

$$\pi_{it} = \mu_{t+1} v_{it} - 1, \tag{8}$$

where the markup, μ_{t+1}, is each firm's sufficient statistic characterizing the aggregate state of the economy:

$$\mu_{t+1} = AP_t = A^{1-\gamma} \left(\sum_{i=1}^{K_t} v_{it} \right)^{-\gamma}. \tag{9}$$

All the aggregate information a firm can benefit from is captured by the aggregate level of markups (9). For tractability we abstract from endogenous variations in the entry margin and focus on the exit decision[10]. We assume that on average λ new firms arrive every period. Because this number is not round, in practice we draw the actual number of entrants from a Poisson distribution with parameter λ:

$$f(k, \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}. \tag{10}$$

New entrants receive the highest possible value of taste of 1. During the life of a firm the evolution of taste, $v_{i,t}$, for its product is described by the following curvature function and transition rule:

$$v_{it} = e^{-g x_{it}}, \tag{11}$$

$$x_{i,t} | x_{i,t-1} = \begin{cases} x_{i,t-1} + \Delta, & 1-\varphi \\ U[0, \bar{x}], & \varphi \end{cases}, \tag{12}$$

where g is a scale parameter and x_{it} denotes the distance of firm i from the frontier. As new firms enter the market, the distance, x_{it}, of an existing firm from the frontier increases with a drift parameter, Δ, which is related to the number of entrants, λ, through the entry rate, s:

$$s = \frac{\Delta}{\bar{x}} = \frac{\lambda}{\bar{K}}, \tag{13}$$

where $\bar{K}$ denotes the average number of firms.

The transition rule in equation (12) captures the idea that entry of λ new products makes older products less desirable, shifting down consumers'

[10] We could determine the number of entrants endogenously by introducing a cost of creating a new production unit like in Caballero and Hammour (1996) equalizing it with the expected value of future profits that unit would entail. Then we would calibrate the elasticity of this cost function to match the empirical amount of fluctuations in the number of entering firms. Since the entry rate of new firms does not vary much over the business cycle in the U.S., this would complicate our analysis without any substantial gain in intuition or any noticeable changes in the behavior of the economy.

relative taste for them by Δ. To make the distribution stationary, we assume that each firm can innovate and can get its product back in fashion with probability φ, in which case the taste for the product is drawn from a uniform distribution. A convenient property of this transition rule is that it preserves the shape of the distribution. In a stationary distribution, tastes for products are also uniformly distributed on the interval $[0, \bar{x}]$ for all values of parameters. Parameter $1 - \varphi$ is related to the persistence of idiosyncratic tastes.

We assume that each firm has an option to exit the market at any period in time. In this case the firm's product loses its appeal forever. For simplicity we assume that the firm never reenters the market and receives a continuation value of zero. The timing of events is as follows: 1) new entrants arrive; 2) nature determines tastes; $\{v_{it}\}$; 3) firms process information and form expectations; 4)based on this information each firm makes a decision whether to stay in the market or exit; 5) the combination of exit decisions determines aggregate variables and profits of individual firms in general equilibrium according to equation (9).

This timing implies that exit decisions are made simultaneously by different firms, so the choices of other firms are not yet known when a firm makes its own exit decision. Because of this timing structure, last period's markup, μ_t, is the contemporaneous aggregate state of the economy. When making the exit decision, each firm uses available information, $I_{i,t}$, to solve the dynamic optimization problem:

$$V_i\left(\mu_t, v_{i,t}\right) = \max_{e_{i,t} \in \{0,1\}} \left\{ E_t \left[\mu_{t+1} v_{i,t} - 1 + \beta V\left(\mu_{t+1}, v_{i,t+1}\right) | I_{i,t} \right], 0 \right\} \tag{14}$$

Each firm maximizes its expected discounted sum of profits by deciding whether to exit or stay and by choosing the information set which determines the way each firm forms expectations. We now turn to the key element of our model: the information structure.

3.2 Information Structure

In this subsection we contrast two versions of the problem of the firm, described by (14): one under full information and another where firms face information processing constraints. The way in which information is acquired and processed by firms has a non-trivial effect on the aggregate behavior of the model. As we shall see, when full information is available the economy is permanently in steady-state and does not exhibit cyclical patterns. We consider this the benchmark scenario. We then compare a model with information processing constraints to this benchmark.

In the model with full information all variables $\{v_{i,t}, \mu_t, v_{-i,t}\}$ are known to all firms. However, not all of this information is useful to the firms. Recall that the aggregate markup, μ_t, is a sufficient statistic which incorporates information on idiosyncratic shocks of other firms and their number through equation (9). Therefore, if firms know the law of motion of the aggregate state $\mu_{t+1}|\mu_t$ they can forecast the aggregate behavior of the economy based on just two variables $\{v_{i,t}, \mu_t\}$ [11]. All the information firms need to know to make the right choices is summarized by these two variables.

Thus, in the full information model firms choose an exit rule $e_{i,t}\left(\mu_t, v_{i,t}\right)$, which solves problem (14). They form rational expectations about the future markup μ_{t+1}, using the true law of motion $T\left(\mu_{t+1}|\mu_t, v_{i,t}\right)$. We solve for the fixed point of the mapping between the law of motion $T\left(:\right)$ and the exit rule $e_{it}\left(\mu_t, v_{i,t}\right)$ by recursive simulation and value function iteration. We obtain a dynamic equilibrium that is characterized by a virtually constant number of firms operating each period, and the economy stays in this steady-state forever. That is because under full information in a dynamic equilibrium firms can perfectly calculate when is the best time to exit, and their choices lead to an optimal outcome.

To model information processing constraints we use the concept of rational inattention. Rational inattention (Sims (2003)[12], (2005), (2006)) blends information theory and economics. The basic idea is to impose a technological constraint on the amount of information a firm can process per unit of time and derive the implications of this assumption for that firm's behavior. Rather than explaining the conceptual foundations of rational inattention,[13] the goal of this section is to provide an informal description of the model (Section 3.3) and to map formally the intuition into our framework (Section 3.4).

In the model with inattentive firms, we constrain the ability of the firm

[11] Simulations show, that incorporating extra information on $v_{-i,t}$ does not have a noticeable effect on the behavior of the model under full information. To save space we do not report these simulations here.

[12] The bulk of the idea of *rational inattention* can be found in C. Sims' 1988 comment in the Brooking Papers on Economic Activity .

[13] See Sims (2003) for an introduction to rational inattention.

to process information which is still freely available. In this case the state variables of the firm $(v_{i,t}, \mu_t)$ are unknown, but a firm can pay the cost of processing information about these variables. Each firm can choose to explore information on aggregate conditions by paying attention to a signal of aggregate profitability $\hat{\mu}_t$. Each firm can also run a market survey asking consumers about their attitude to product i to get a signal about consumer tastes, $\hat{v}_{i,t}$. Firms use Shannon's channel to acquire and process these signals. Whether to pay attention to signals about the aggregate and idiosyncratic component of demand, and what to look for in each signal, is the choice variable of each firm. As before, each firm forms rational expectations about the aggregate markup and the taste for its own product using the true law of motion. A firm keeps track of a probabilistic perception of both variables, which it optimally updates using the information it chooses to receive over time.

3.3 Shannon's channel in our model

Consider a firm that wants to time its exit. This firm does not know exactly the conditions of the market in which it operates. In particular, the firm does not know the details of its own demand in terms of economy-wide markup, μ, and consumers' preference for its own product, v_i. Let $S_i \equiv \{\mu, v_i\}$ represent the source of uncertainty for firm i.

Firm i can choose a signal about S_i, but knowing each and every detail about S_i is beyond the firm's ability to process information. For instance, it would require a massive amount of time and resources to gauge the interest in product i by surveying each and every potential buyer of the product in order to reduce uncertainty in v_i. By contrast, firm i might find it optimal to pull a small sample of individuals -say a "focus group"- and base its estimate of $\hat{v}_i$ on this sample evidence.

Similarly, the firm can acquire signals about the markup μ using news reports, publications of statistical agencies and other public or private sources of information about market conditions. The firm is not capable of processing all of that information, but it can choose a few sources on which to base its estimate of $\hat{\mu}$. The firm can choose to acquire more information about $\hat{\mu}$ than about $\hat{v}_i$ if it thinks it is profitable to allocate more attention to the aggregate component than the idiosyncratic one, or vice versa. As a result, in choosing a signal for S_i, the firm balances the trade-off between informativeness and precision of the signal subject to the limits imposed by its information-processing and budget constraints.

Based on the observed signals, each firm optimally decides whether to operate or exit the market. If the firm decides to stay, it operates and receives profits π_i. If it decides to exit, the firm does not have the opportunity to enter the market ever again. Each period the firm chooses signals based on earlier profit and signal realizations which are all summarized by the firm's perception $g(S_i)$ of the state of the economy at the beginning of period t. A firm continues to run its business in this fashion up until the period it decides to exit.

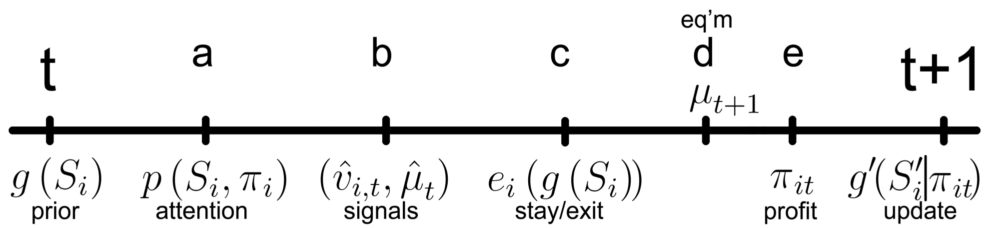

FIGURE 3. **Model Timing**

Such a story can be directly mapped into a dynamic rational inattention framework using mutual information as the technology that regulates the flow of information that is passed through the channel. At the beginning of period t, the firm does not know its demand, S_i, but it has a prior on it, $g(S_i)$. Before processing any information, the firm's uncertainty about the state is summarized by the entropy of its prior, $\mathcal{H}(S_i) \equiv -E[\log_2(g(S_i))]$, where $E[.]$ denotes the expectation operator.[14] Before processing any information, profits, π_i, are also a random variable.

To reduce entropy, the firm can choose to have a detailed report from a statistical agency or to look at a focus group. The two options differ in the amount of information content and, as a consequence, in effort of information-processing. We assume that the shape of the distribution of attention directed at these two sources of information and their relevance for future profits can be chosen by the firm. We denote the joint distribution of attention to signals and profits $p(S_i, \pi_i)$.

[14]Entropy is a universal measure of uncertainty that can be defined for a density against any base measure. The standard convention is to use base 2 for the logarithms, so that the resulting unit of information is binary and called a *bit*, and to attribute zero entropy to the events for which $p = 0$. Formally, given that $s \log(s)$ is a continuous function on $s \in [0, \infty)$, by l'Hopital Rule $\lim_{s \to 0} s \log(s) = 0$.

Both types of reports contribute to the reduction in uncertainty about demand by an amount equal to $\mathcal{H}(S_i|\pi_i) = -\int p(S_i, \pi_i) \log_2 p(S_i|\pi_i) \, d\pi_i dS_i$, which is the entropy of S_i that remains given the knowledge of π_i. The information flow, or maximum reduction of uncertainty about the prior on demand, is bounded by the information content of the signal:

$$I(\pi_i; S_i) = \mathcal{H}(S_i) - \mathcal{H}(S_i|\pi_i) \leq \kappa \tag{15}$$

where κ is measured in number of bits transmitted.

We assume that the cost of processing information is linear in capacity: $c(\kappa) = \theta * \kappa$. The firm knows in advance that information has a cost θ and chooses the optimal distribution of attention $p(S_i, \pi_i)$ which equalizes the cost of information about future profits with the potential gain in profits from more precise information.

After receiving signals about the state S_i, firms simultaneously decide whether to operate or exit the market. These exit decisions $e(g(S_i))$ determine the subset of firms which operates in the market in period t, and, thus, through equation (9), pin down the equilibrium markup level μ_{t+1}, which in turn, through equation (8), determines profits π_i. Firms do not observe the markup, but can observe profits. At the end of period t each operating firm uses the signals and the realized profits, π_i, to update its prior on demand $g(S_i)$ via Bayes' rule. The update $g'(S_i'|\pi_i)$ which incorporates the new information is carried over to the next operating period.

Our example with the focus group and statistical reports illustrates how people handle everyday decision of weighting the effort of processing all the available information -own demand- against the precision of the information they can absorb -details in the report- guided by their interest -maximizing profits-. This setup is the core of rational inattention: information is freely available but people can process it at a finite rate.

The benefit of using Shannon's capacity constraint is that it provides a general measure of uncertainty that depends only on the distributions of the random variables passed through the channel and allows us to abstract from the exact implementation of the information processing constraint. The cost of this approach is that to solve dynamic rational inattention models one needs to solve an infinite-dimensional problem involving current and future distributions of the random variables of interest. Closed form solutions for these kinds of problems are limited to a handful of cases.[15]

[15] See Sims (2003) and Matejka and Sims (2009).

3.4 Rational Inattention model

Here we describe the formal version of the information processing problem sketched in the previous subsection. We assume that each firm knows the process characterizing the exogenous entry of new firms and the law of motion of the state vector, $S_i = \{\mu_t, v_{i,t}\}$, which we characterize using a transition function, $\tilde{T}(:)$. Firms keep track of their perception $g(S_i)$ of the joint distribution of the state vector. Then, firm i solves the value iteration problem:

$$V(g(S_i)) = \max_{e_i(g(S_i))} \{EJ_i, 0\} \tag{16}$$

where

$$EJ_i \equiv \max_{p(S_i,\pi_i)} \int [\pi_i(S_i) - \theta\kappa + \beta V'(g'(S_i'))]\, p(S_i,\pi_i)\, d\pi_i dS_i \tag{17}$$

subject to the information constraint:

$$\kappa = \int p(S_i,\pi_i) \log\left(\frac{p(S_i,\pi_i)}{p(\pi_i)\, g(S_i)}\right) d\pi_i dS_i \tag{18}$$

and the updating rule for perception:

$$g'(S_i'|\pi_i = \bar{\pi}) = \int \tilde{T}(S_i'; S_i|\pi_i = \bar{\pi})\, p(S_i|\pi_i = \bar{\pi})\, dS_i \tag{19}$$

$$g(S_0) \text{ given} \tag{20}$$

Equation (16) is the value function of the firm, which is the maximum between the outside option of zero, if the firm decides to exit, and the expected discounted value of profits (17), if the firm decides to operate. The value function in (17) combines the expected value of profits this period, $\pi_i(S_i)$, and the expected value of future periods, $V'(g'(S_i'))$, discounted at rate β. The maximization is over the joint distribution $p(S_i,\pi_i)$ which is also the metric under which firm i defines its own expectations.

The maximization is constrained by the Shannon's processing capacity, (18), which is a function of the optimal choice of the firm, $p(S_i,\pi_i)$, and the prior $g(S_i)$. The interpretation of this constraint has been discussed in the previous subsection. Here we recall that θ is the shadow cost of processing information associated with capacity κ defined by equation (18).

Equation (19) represents the law of motion of the state $g(S_i)$, i.e. the posterior $g'(S_i')$ updated using Bayes' law. Given a realization of profits,

$\pi_i = \bar{\pi}$, the expression in (19) convolutes the stochastic knowledge of the law of motion of S_i summarized by the transition function $\tilde{T}\,(:)$ with the optimal strategy implemented that had led to $\bar{\pi}$, i.e., $p\,(S_i|\pi_i = \bar{\pi})$. Finally, (20) provides the initial condition of the problem. Additionally, we require the optimal $p\,(S_i, \pi_i)$ to belong to $\mathcal{D}\,(S_i, \pi_i)$, that is the space of all the distributions for which:

$$p\,(S_i, \pi_i) \geq 0,\, \forall \pi_i, S_i \tag{21}$$

$$\int\int p\,(S_i, \pi_i)\, d\pi_i dS_i = 1 \tag{22}$$

$$\int p\,(S_i, \pi_i)\, d\pi_i = g\,(S_i) \tag{23}$$

An equilibrium in this economy is a combination of optimal signals $p\,(S_i, \pi_i)$, an exit rule $e_i\,(g\,(S_i))$, a law of motion $\tilde{T}\,(S_i'; S_i)$, prices $\{p_{it}, P_t\}$ and allocations $\{q_{it}, C_t, l_{it}, E_t\}$ such that (i) signals and exit rules solve the firm's problem (16)-(23) given the law of motion, (ii) allocations are optimal given prices and prices clear markets as described by equations (4)-(9), and (iii) the law of motion is consistent with the combination of firms' choices.

We prove in the appendix, that the problem of the firm is a contraction mapping, hence, it has a unique solution given the law of motion. Any solution to the problem of the firm maps uniquely into allocations, prices and a law of motion. Given this, we expect the equilibrium to exist and be unique.[16]

To find an equilibrium we solve for the fixed point of the tuple: $\{\tilde{T}\,(S_i'; S_i)\,,\, p\,(S_i, \pi_i)\,,\, e_i\,(g\,(S_i))\}$, such that the law of motion $\tilde{T}\,(:)$ is the outcome of exit decisions $e\,(:)$ based on the attention allocation solution $p\,(:)$, and the attention allocation is optimal given the law of motion. We approximated the law of motion using a first-order Markov chain. For a detailed description of a pseudo-code which we used to find the equilibrium see the Appendix.

Iterations between the solution of the firm's problem and simulations of the economy show that convergence to a fixed point is relatively quick. Moreover, our confidence in the existence and uniqueness of a fixed point in practice is reassured by the fact that significant variations in starting points for the law of motion do not lead us to different equilibria.

[16] Though all the variables are defined on a compact support, decision rules are not necessarily continuous because the exit decision is discontinuous. Because of this, we were unable to prove existence or uniqueness of the equilibrium in general.

Note also, that the problem of the firm without information processing constraints described by equation (14) is a special case of the constrained version when $\theta = 0$. Therefore, the information processing constraint is the only source of any differences between the two models we consider. We first calibrate and explore the quantitative behavior of two versions of the model in section 4, and then lay down the intuition behind the effects of information processing constraint on the equilibrium outcomes in section 5.

4 Results

4.1 Calibration

Jaimovich (2007) discusses in detail how to calibrate a similar model to match closely second moments of the data and emphasizes the fact that this calibration comes at the cost of descriptive realism in several dimensions. The main concerns are related to the interpretation of sectors of the economy and to accounting for variations in firm sizes. Gabaix (2011) shows that because of the extremely fat tailed distribution of firm sizes in the U.S. economy, idiosyncratic fluctuations at the firm level do not wash away in the aggregate. This makes his granular framework much more suited for our analysis of an economy populated by a finite number of firms than Jaimovich's specification with a continuum of sectors.

The focus of our exercise is on business cycle asymmetries, so we are mostly interested in skewness and correlations. Since these second and third moments are normalized and do not decay with the law of large numbers, we do not attempt to follow either Gabaix's or Jaimovich's calibrations targeted at second moments. This makes our exercise much simpler and results significantly more transparent. For a more detailed discussion we refer the reader to section 6.

Each time period is a quarter. This choice determines the discount factor, β, at 0.99 and the entry rate, s, at 5%, the average fraction of opening establishments among total private sector establishments in the U.S. in a given quarter (as measured by BED). We set the curvature of utility γ close to unity, which implies logarithmic utility, consistent with a balanced growth path. We fix the grid size for $x_{i,t}$ to the unit interval and set $\bar{x}$ at 0.9. We set the scale of the idiosyncratic component of tastes g to 0.8, which implies an average markup of 90% - the mean of the marginal price-cost markup in the U.S. economy over the last 50 years (see Nekarda and Ramey (2010)).

We set the probability of innovation, φ, to 0.8, which in our view captures well the dynamic and unpredictable nature of tastes for particular products. We set this parameter in the ballpark of the numbers from Cooper, Haltiwanger and Willis (2007) who use establishment-level data on employment, hours, wages and profitability, as well as the distribution of employment growth at the producer level to estimate costs of adjustment at the job creation and job destruction margins. They estimate the autocorrelation of establishment-specific profitability shocks to be 0.33, while the standard deviation of these shocks to be as large as 0.23. [17]

We are not aware of direct evidence which would allow us to pin down the shadow cost of information, θ. We set the cost of information to 0.01, which implies that the total shadow cost of information varies in the range from 10% to 20% of average profits in the dynamic equilibrium. In section 6 we do robustness checks by exploring how the behavior of the model depends on these parameter choices.

Finally, we set production capacity, A, targeting the average number of firms at 15. We are forced to set such a low number of firms due to computational constraints placed by the complexity of the computational procedure. We later discuss the consequences of this choice and conclude that our results are not significantly undermined by this assumption.

We set the grid for the markup, μ_t, to equispaced intervals between 1.15 and 2.77. Because of the computational intensity of the model with inattention, we use a relatively coarse 20 point grid. We set the length of simulations to 250 periods, from which we discard the first 50. Thus, the total history from which firms can learn is comparable to the length of available US data.

In each case we solve for a fixed point of the mapping between the exit rule $e_{i,t}(:)$ and the transition rule $T(:)$. We switch recursively between finding the solution of the problem of the firm by value function iteration, and simulating the model using the solution to obtain the law of motion. Table 5 summarizes the calibration for the numerical algorithm. Note that for the information constrained model the joint distribution $g(v_i, \mu)$ has been constructed so that the points on the simplex have marginal mean and standard deviation

[17] We think that indirect estimates of Cooper, Haltiwanger and Willis (2007) are best suited for our calibration because they cover firms in all sectors of the economy and are computed at quarterly frequencies. In contrast, the best direct estimates of persistence of idiosyncratic shocks provided by Foster, Haltiwanger and Syverson (2008) are only available for a restricted subset of manufacturing goods at quinquennial frequencies.

that reflect the ones from the empirical distribution. The transition function then convolutes the transition properties of $g(\mu, x_i)$ for each possible value of profits to assign a distribution for next period values of the states, $T(.) \equiv T(\mu', v_i' | \mu, v, \pi)$.

Symbol	Parameter	Value
β	Time discount factor	0.99
γ	CRRA coefficient	0.95
g	Scale distance	0.8
s	Entry rate	0.05
φ	Probability of innovation	0.8
θ	Cost of processing information	0.01
π_i	Grid for profits	$[-0.48, 1.78]$
μ	Grid for markup	$[1.15, 2.77]$
x_i	Grid for distance	$[0.0, 1.0]$

Table 5. **Numerical approximation.**

It is important to note that the solution of the model is extremely computationally intensive. Even using advanced programming techniques on a powerful computational cluster, a solution for a single calibration of the overly simplified model with a relatively small number of firms, takes about a week. Thus, the computational intensity places significant restrictions on the scope of our analysis. To explore the sensitivity of our mechanism to the modeling assumptions we make we do robustness checks by varying several key parameters and find that some of them have a nontrivial effect on the behavior of the model. Incorporating further complications, such as financial and labor market frictions, as well as a more precise calibration of the model would entail a significant computational effort, which we leave for future research.

4.2 Simulations

To compare the behavior of the information constrained economy with the full information model we first plot the paths of output and markups, entry and exit rates for sample 50 year periods for each of the two models. Here we adopt a simple definition of a recession - an event when the number of firms drops by at least 20 percent in a single period. The top panels of Figures 4A and 4B below show the simulated paths with shaded recessions.

The difference between the two models is apparent. The full information economy is characterized by symmetric fluctuations that resemble white noise, without any apparent asymmetry between exit and entry rates. Recessions are rare. The inattentive economy alternates between periods of slow expansion, characterized by accumulation of capacity and employment, and sharp downward adjustments. These adjustments are characterized by large bursts of firm exit and job destruction, sharp decreases in output and employment, sharp increases in profit margins. The path of the economy is characterized by a large asymmetry between sharp busts and long and persistent booms.

The bottom panels of Figures 4A and 4B compare the laws of motion of the aggregate markup for the two models. Figure 4B demonstrates, that in an inattentive economy the aggregate markup has two potential behaviors. It either drifts down slowly, or jumps up sharply. Outliers above the diagonal are more common than below the diagonal. In the full information economy, off-diagonal behavior is much less pronounced.

To compare the statistical properties of the two models we compute standard deviations of output and markups, autocorrelation of output, average periods between adjustments of different sizes, skewness of growth rates of output, and markups, of exit and entry rates. Table 6 summarizes these statistics for the two models.

The main message to take away from Table 6 is that in the presence of an information constraint, the economy exhibits pronounced cyclical behavior. The economy is more volatile, cycles are more asymmetric, larger contractions are at least as common as small contractions, the skewness of exit rates increases compared to entry rates. Since the only difference between the two models is the information constraint, ithis constraint is the source of the asymmetry. Here we provide a qualitative description of the intuition behind this result and refer to the next section for a detailed explanation.

Every period inattentive firms observe profits and need to distinguish between two potential sources of variations in profits. Knowing both the aggregate and the idiosyncratic component in the full information model is enough to understand exactly when to exit. This is not the case when processing information is costly.

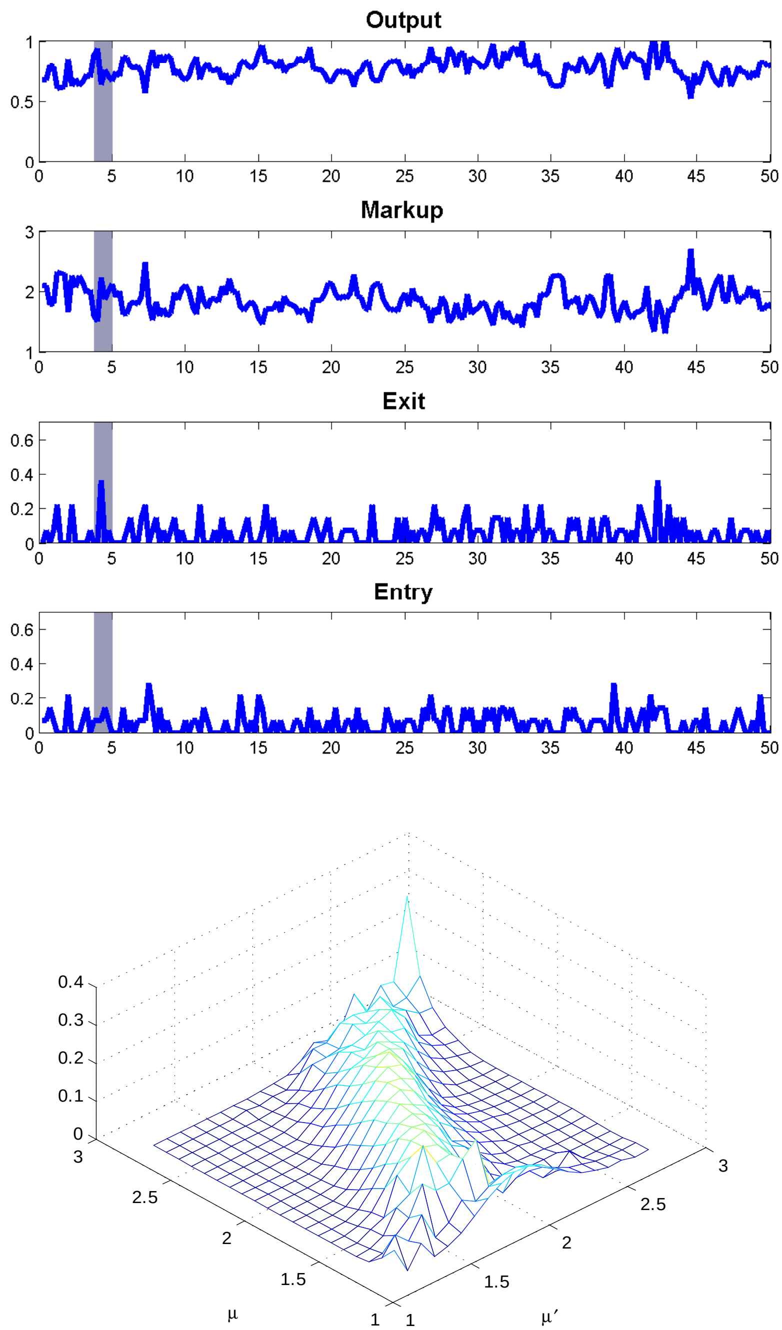

FIGURE 4A. **Model with Full Information.**

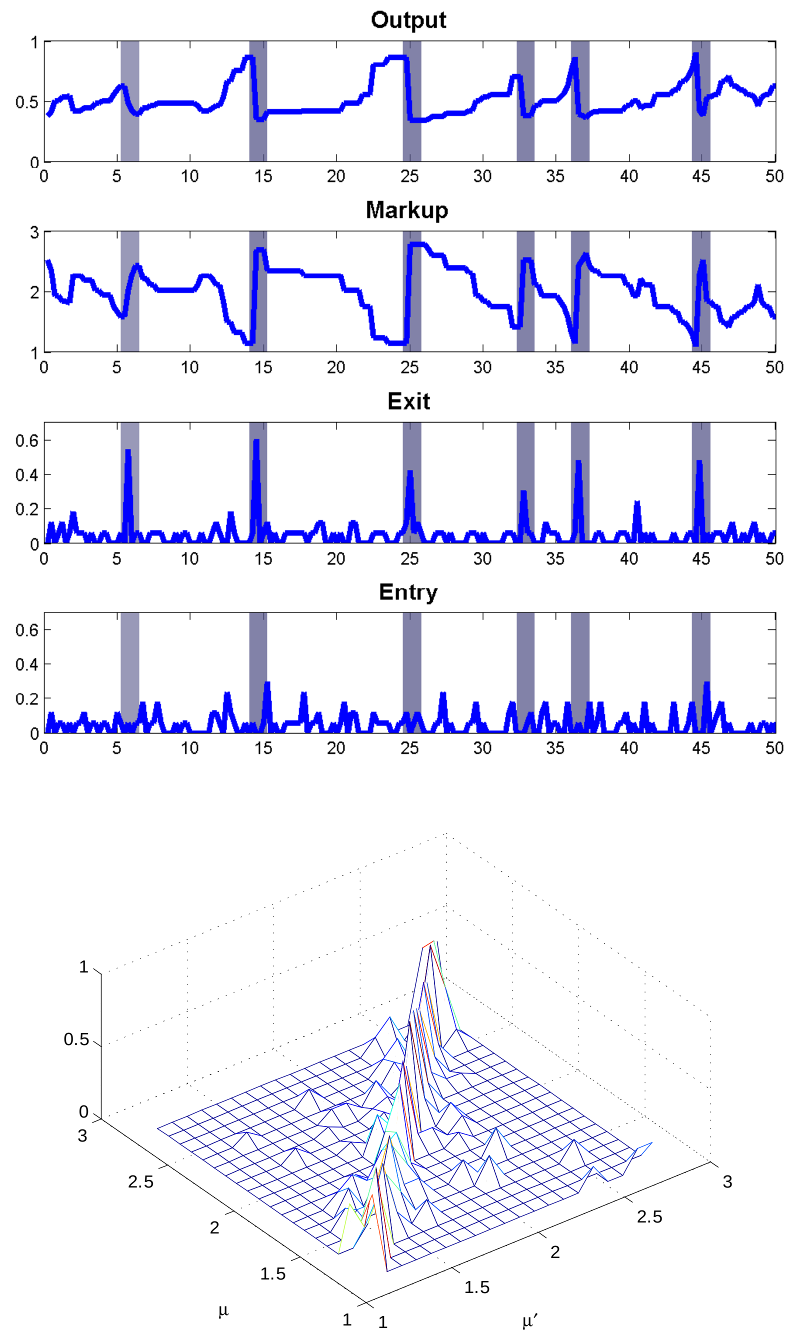

Figure 4B. **Model with Inattention.**

	σ_Y	σ_P	ρ_Y	$\tau_{5-10\%}$	$\tau_{10-20\%}$	$\tau_{>20\%}$	$\gamma_{\Delta Y}$	$\gamma_{\Delta P}$	γ_X	γ_R
Full Info	0.13	0.13	0.57	1.8	2.6	15	-.18	.26	1.6	1.1
Inattention	0.26	0.21	0.79	9.4	10	8.3	-4.0	3.8	4.0	1.6

Table 6. **Model comparison**

σ denotes standard deviation; ρ - autocorellation; $\tau_{\%}$ - average periods between contractions;
γ - skewness; ΔY - growth rate of GDP; ΔP - growth rate of markups; X - exit rate; R - entry rate.

In good times (when markups are high) none of the firms needs to exit, so firms do not need to pay much attention to any of the signals. Over time, as more firms enter, conditions start to worsen (markups fall), so firms would like to pay more attention to changes in the economy. They are unable to do this, as the relative cost of attention (compared to profits) increases when markups fall.

Out of the two dimensions of the signal, firms choose to focus more of their attention on the aggregate dimension. This is because the aggregate signal is more persistent, so less capacity needs to be used to acquire information. This makes the aggregate signal relatively cheaper in bad times. It is also more valuable, as it is a better predictor of a coming recession.

Because attention is costly, it takes a long time for the firms to realize that times are bad in the aggregate and to consider exit a plausible option. When there is so much uncertainty, each firm needs a clearer perspective to make the exit decision, and that takes time. As a result, firms tend to delay their exit decision, which drives markups and profits even lower. When firms finally realize that the aggregate markup is indeed low, that also gives them an idea of how good they are relative to other firms, and helps them decide whether to exit.

Since markups are already very low at the moment when firms realize it, and firms have been coordinated to similar perceptions of the economy by paying attention to correlated signals on aggregate conditions, many firms choose to exit simultaneously. As a consequence, the markup rises sharply and stays high for a considerable period of time.

5 Rational Inattention and Coordination

5.1 The Mechanism

To understand the mechanism better in this section we study in detail the individual decisions of firms. In every period firms can choose to pay attention to a signal on aggregate conditions and a signal on tastes. In this section, we focus on which pieces of information firms choose to pay relatively more attention to, how this new information affects their perceptions of the state of the economy and how it affects their exit decisions.

First we show that the effect of the information friction is amplified by variations in the relative cost of information. We show that times of high profit margins are times when information is less costly relative to profits. On the flip side, when firms need to pay most attention to market conditions, is when profits are lowest, and hence the relative cost of attention is highest. Another consequence of this trade-off is that firms pay more attention to both signals in good times compared to average times. That is because information is especially cheap relative to profits when profits are high.

To illustrate these findings, we compute capacities spent on processing information about markups, $\kappa_\mu|\pi_i$, and about tastes, $\kappa_{v_i}|\pi_i$, both conditional on profits. Figure 5 shows how capacity is allocated between markups and tastes for five particular values of profits: $\pi = (\{-0.05\}, \{0\}, \{0.05\}, \{0.35\}, \{1\}, \{1.5\})$. The picture displays the average capacity used to understand tastes (blue solid line) and markups (green dashed line) in the optimal solution $p^*(S_i, \pi_i)$, where the average is taken over all priors $g(S_i)$ in the simplex and markups and tastes respectively are integrated out.

Figure 5 illustrates three patterns. First, because uncertainty and persistence of tastes do not change over time, the capacity necessary to process one bit of information is the same independent of profits. However, the costs of processing information when compared to profits are smaller in good times. Consequently, firms allocate more capacity (pay more attention) to tastes when information is relatively cheaper (in good times).

Second, while information about the aggregate markup is relatively cheaper in good times, it is also more valuable in bad times. This is because in bad times firms need to make an important exit decision, which makes precise knowledge of the aggregate state of the economy more valuable. This pattern of changes in both the value and relative cost of information is the reason for the V-shaped behavior of capacity allocated to the aggregate markup.

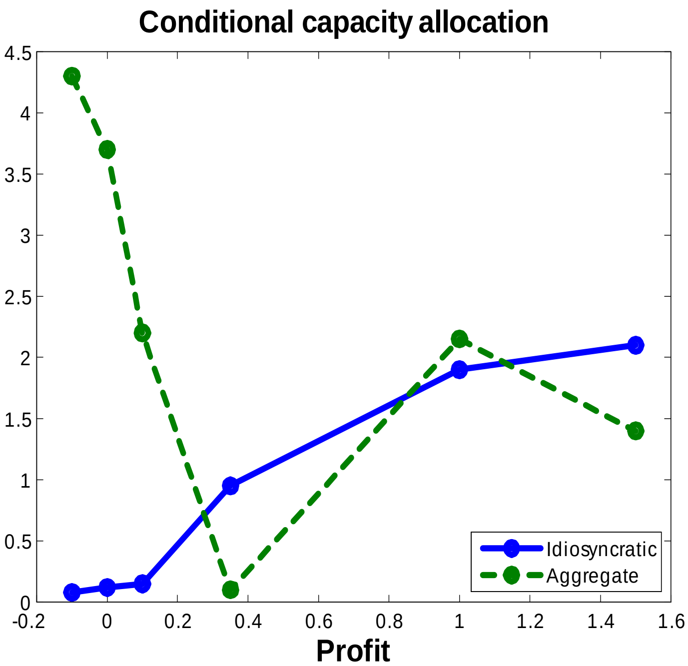

FIGURE 5. **Capacity given profits, π**

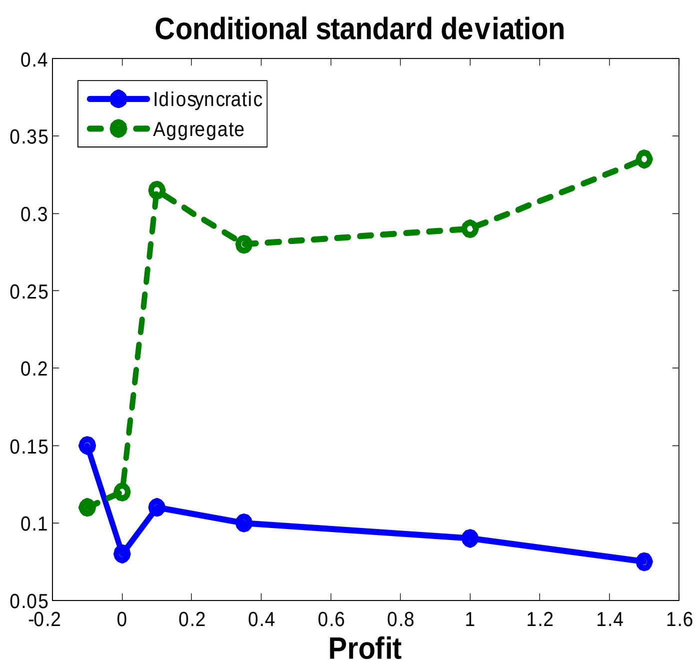

FIGURE 6. **Standard deviation of aggregate markup, μ, and taste shocks, v_i conditional on profits, π**

Finally, information about the aggregate markup is generally more valuable and less costly to the firm than information on tastes. Therefore, more attention is allocated to aggregate than to idiosyncratic variables, especially in bad times.

This asymmetric allocation of attention leads to an asymmetry in precision of the perception of aggregate and idiosyncratic variables. To illustrate this finding, we compute the perceived variance of the signal conditional on a particular value of profits $\pi_i \in \Omega_\pi$. That is, let $E^*_{\pi_i}$ denote the optimal distribution conditional on a particular π_i. Then the variance of each signal conditional on profits is computed as follows:

$$\hat{\sigma}^{\pi}_X \equiv \sqrt{Var\left(X|\pi_i \in \Omega_\pi\right)} = \sqrt{E^*_{\pi_i}\left(X^2\right) - \left(E^*_{\pi_i}\left(X\right)\right)^2}$$

for $X = v_i$, μ. Figure 6 displays the precision of each signal conditional on profits.

Figure 6 illustrates that more attention devoted to the aggregate signal in bad times makes it more precise in bad times, while less attention devoted to tastes makes them less precise. Note how conditional on $\pi = -0.05$, the standard deviation of markup μ is lower than in other cases. This is due to the fact that the firm is paying much more attention to the aggregate markup when profits are low. This choice tends to reduce the volatility of the optimal markup signal. The opposite is true for the standard deviation of v_i for low profits.

When profits are high, firms do not pay as much attention to markups, which is reflected in a high volatility of the optimal price signals. Likewise, firms pay relatively more attention to the signal on v_i, so the optimal perceived volatility of v_i is reduced. When profits are in the middle, the overall information flow acquired by the firm is smaller than for the two other cases resulting in a smaller reduction of uncertainty of the economic environment as a whole. Nevertheless, the total information flow makes the relative volatility of the signal for v_i smaller than for μ.

Because information about markups is so valuable in bad times, and information is overall more costly in bad times, firms tend to focus most of their attention on aggregate variables when their profits are low. In addition to reducing volatilities, firms can learn about tastes through their knowledge about markups using the direct observation of profits. Instead of reducing uncertainty of tastes directly, firms can infer tastes by increasing precision of the aggregate signal and by correlating it with the signal on tastes.

If the markup is low, but profits are relatively high, chances are that tastes are high too. We illustrate this logic in Figure 7 which shows the cross correlation between μ and v_i conditional on profits, computed as follows:

$$\rho^*_{\pi_i} \equiv \frac{Cov\left(v_i, \mu | \pi_i\right)}{\hat{\sigma}^{\pi}_{v_i} * \hat{\sigma}^{\pi}_{\mu}} \tag{24}$$

The strong negative correlation between the two signals conditional on low profits shows that firms tend to rely solely on aggregate variables both to infer the aggregate state and to learn about tastes in bad times.

Thus, signals about aggregate variables play the role of a coordination device in our model. Joint decisions of firms to focus most of their attention on the same signals lead to a correlation in their exit decisions. One reason why the endogenous choice of signals is important in our model is that we allow firms to choose both precision and correlation of the two signals conditional on perceptions. This choice is the main driving force behind the coordination mechanism and asymmetric fluctuations.

To summarize, we find that paying attention to both aggregate and idiosyncratic information becomes relatively more costly as profit margins fall. This makes monitoring aggregate conditions harder and leads to an aggregate reduction in attention as the economy expands. We find that firms increase their share of attention allocated to monitoring aggregate conditions when increased competition lowers average profit margins. This is partly because when a firm knows its profits, precise information about just one of the signals is enough to identify the other one. Accordingly, unless profit margins are high, firms choose to pay most of their attention to the signal on aggregate conditions because it is more persistent, and, hence, less costly to identify.

The last element we want to illustrate is the condition that needs to be met for a firm to decide to exit. The exit decision is based on the perception that the product has gone out of fashion. As depicted in Figure 8, the probability of exit increases sharply with the distance from the frontier, x_i (on the horizontal axis). Firms base their decision on their perception of this idiosyncratic component. They wait not only for the aggregate markup to fall low enough, but also for the product to go out of fashion. Hence, they choose a cutoff value of the perception such that an increased probability of exit does not immediately result in a high probability of staying due to misperception (this corresponds roughly to $x_i = 0.75$).

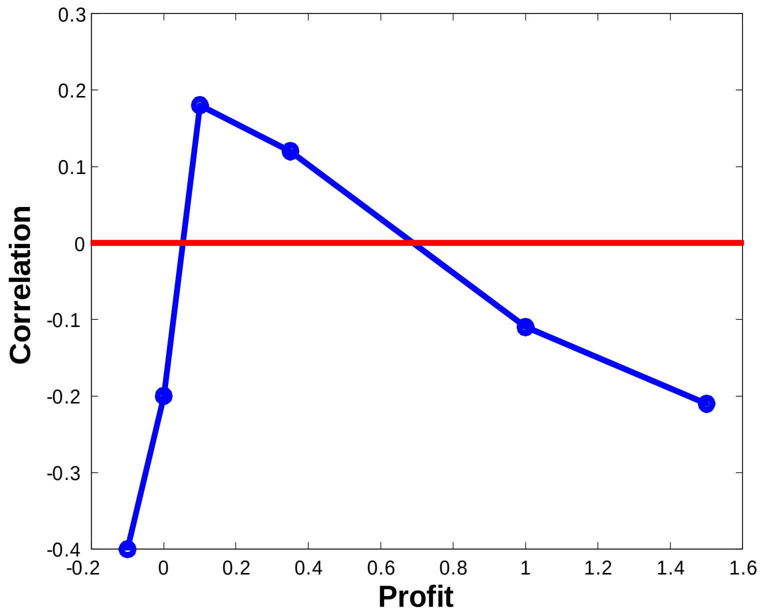

Figure 7.**Correlation of markup μ, and taste shocks, v_i, conditional on profits, π.**

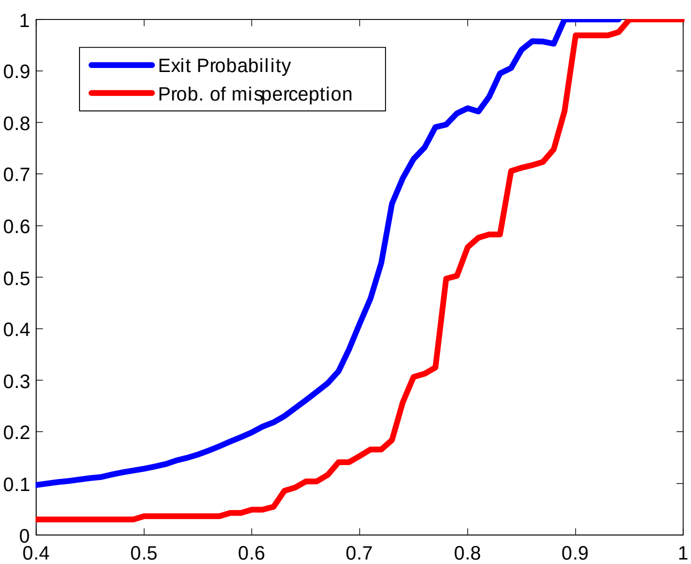

Figure 8. **Probabilities of exit and misperception by distance to the frontier, x_i**

The probability that firms perceive their products as out of fashion are highly correlated due to the fact that they are all based on observations of the same aggregate signal $\hat{\mu}$. The perception that the product is out of fashion moves from a 30 percent to a 80 percent probability very quickly, so a large fraction of firms decide to exit simultaneously. Because firms base their perceptions on a common signal, the conditions which trigger exit tend to occur simultaneously for many firms.

Figure X in the appendix illustrates the workings of the described perception-based mechanisms all together. Expansion episodes are characterized by entry of new firms, which leads to decreases in aggregate markups. This lowers profits and makes paying attention more costly. As a result, information processing capacity κ falls. Firms allocate most of this capacity to the aggregate signal, so the precision of their perception of the aggregate signal becomes sharper. It helps them identify the idiosyncratic signal through induced negative correlation with the aggregate signal.

Similarity of the problem being solved and common information used to make the exit choice lead to coordinated exit. A large number of firms end up making a simultaneous decision to exit, which levels the playing field for survivors, rising their profit margins and starting the cycle all over again.

6 Discussion

In this section we first explore the effects various changes in parameters have on our results. Then, we discuss potential implications of a significant increase in the number of firms for the cyclical behavior of our model. In light of these implications, we compare the predictions of our model to the data and discuss its explanatory power.

6.1 Sensitivity to Key Parameters

First, we explore the sensitivity of our results to variations in two important parameters: the entry rate, s, and the probability of innovation, φ. Table 7 displays the behavior of the two models under the benchmark calibration ($s = 0.05, \varphi = 0.8$) and compares it to alternative calibrations ($s = 0.03$ and $\varphi = 0.2$).

When full information is available, the decrease in the entry rate, s, makes cycles slightly more asymmetric, while persistence of individual histories,

$1-\varphi$, increases aggregate persistence without affecting much the length and asymmetry of the cycle. When capacity of processing information is limited, the cycles are much bigger and much more asymmetric compared to the full information case.

In this context, both a decrease in the entry rate s and an increase in persistence of individual histories (reduction of φ) help alleviate uncertainty, reducing the amount of information that needs to be processed. A lower entry rate makes the aggregate component more predictable, while persistence of individual histories makes the idiosyncratic component more predictable. As a result, less effort is required to process information, and better coordination is achieved.

Second, since the main difference between the two models we consider is the increase in the cost of processing information, θ, from 0 to 20 percent of average profits, it is instructive to explore the effect of a further increase in the cost of information processing. Surprisingly, an increase by a factor of five in the cost of processing information, which now accounts for about 80 percent of average firm profits, leads to a decrease in both the asymmetry and persistence of cycles. The reason for this is that in this case the costs of processing information become so high, that firms give up on getting a precise signal even about aggregate conditions, and base their exit decisions only on profits in the previous period. This behavior leads to almost uniformly distributed random exits of firms which are not necessarily the ones producing the most outdated products.

Table 8 shows how the degree of asymmetry depends on various other changes in parameters. Results reported in Table 8 confirms the hump-shaped response of skewness to variations in the cost of information, as well as the reduction in skewness associated with increased predictability of (reduction in uncertainty about) idiosyncratic shocks and entry rates.

6.2 Number of Firms

Another important concern with the calibration of the model is the number of firms used in the simulation. To convince the reader, that the assumption of 15 firms is not a major problem for our findings, we discuss the effect this assumption has on the main statistics of interest.

First, note that in a granular economy discussed at length by Gabaix (2011) even without coordination mechanisms aggregate fluctuations can be a result of idiosyncratic fluctuations at the firm level if the distribution of firm sizes has fat tails. When the distribution of firm sizes is Pareto, the speed of decay of idiosyncratic fluctuations is $\ln N$ instead of $N^{1/2}$. This makes a huge difference, as 10^6 firms in a world with a symmetric size distribution would be equivalent to an economy with on the order of 10^2 firms in a world with a Pareto firm size distribution.

	σ_Y	σ_P	ρ_Y	$\tau_{5-10\%}$	$\tau_{10-20\%}$	$\tau_{>20\%}$	$\gamma_{\Delta Y}$	$\gamma_{\Delta P}$	γ_X	γ_R
Full Info	0.13	0.13	0.57	1.8	2.6	15	-.18	.26	1.6	1.1
$s = 0.03$	0.12	0.13	0.65	2.2	5.1	62.5	-.39	.29	2.6	1.9
$\varphi = 0.2$	0.13	0.13	0.67	2.2	2.8	7.8	-.42	.58	2.3	1.0
Inattention	0.26	0.21	0.79	9.4	10	8.3	-4.0	3.8	4.0	1.6
$s = 0.03$	0.15	0.13	0.84	1.8	6.3	9.4	-1.7	0.6	3.6	1.6
$\varphi = 0.2$	0.13	0.11	0.49	1.8	2.4	11	-.08	.31	1.6	1.3
$\theta = 0.05$	0.13	0.12	0.52	1.5	24	19	-.01	.03	1.1	1.2

TABLE 7.**Sensitivity to variations in parameters.**

σ denotes standard deviation; ρ - autocorellation; $\tau_{\%}$ - average periods between contractions; γ - skewness; ΔY - growth rate of GDP; ΔP - growth rate of markups; X - exit rate; R - entry rate.

s	θ	φ	$\gamma_{\Delta Y}$	$\gamma_{\Delta P}$	γ_X	γ_R
5%	1%	0.8	-4.0	3.8	4.0	1.6
3%	1%	0.8	-1.7	0.6	3.6	1.6
3%	1%	**0.65**	-1.1	1.0	2.0	1.3
5%	1%	**0.2**	-.08	.31	1.6	1.3
5%	**5%**	0.8	-.01	.03	1.1	1.2
5%	**0.5%**	0.8	-.41	.34	1.3	1.1
3%	**2%**	0.8	-1.2	0.95	2.0	1.3

TABLE 8. **Sensitivity of skewness to variations in parameters**

s denotes entry rate; θ - cost of processing information; φ - probability of innovations; γ - skewness; ΔY - growth rate of GDP; ΔP - growth rate of markups; X - exit rate; R - entry rate.

	σ_C	σ_P	ρ_C	$\tau_{5-10\%}$	$\tau_{10-20\%}$	$\tau_{>20\%}$	$\gamma_{\Delta Y}$	$\gamma_{\Delta P}$	γ_X	γ_R
K=15	0.13	0.13	0.57	1.8	2.6	15	-.18	.26	1.6	1.1
K=35	0.09	0.09	0.59	2.9	8.1	∞	-.13	.15	1.1	0.7
K=70	0.07	0.07	0.64	5.3	∞	∞	-.05	.08	0.5	0.6
K=100	0.05	0.05	0.62	7.1	∞	∞	-.14	.17	0.5	0.4

TABLE 9.**Sensitivity to the number of firms, K.**

σ denotes standard deviation; ρ - autocorellation; $\tau_{\%}$ - average periods between contractions; γ - skewness; ΔY - growth rate of GDP; ΔP - growth rate of markups; X - exit rate; R - entry rate.

This implies that we can get a good idea of the behavior of our informationally unconstrained economy in a granular world by increasing the number of firms to 100. Table 9 shows what happens when we gradually increase the number of firms. Second moments of fluctuations gradually decrease to values, which are similar to those observed in developed countries. The length of cycles increases to values, which are much closer to the average lengths of cycles in developed countries.

Note also, that the third moments targeted at characterizing the asymmetries of the fluctuations are determined largely by the properties of the Poisson entry process. As the asymmetry of this process declines with the increase in the number of firms, so does the exit rate. This implies that our full-information model produces fluctuations of the magnitude which is in the ballpark of actual fluctuations, but do not come even close to explaining their asymmetry.

The second important point we make is that skewness is a normalized variable, which does not decay with the law of large numbers if you aggregate idiosyncratic decisions of firms. Using the same method Gabaix used to derive properties of standard deviations, we derive properties of skewness in the appendix. This derivation shows, that the number of firms has a very limited effect on the asymptotic skewness of GDP.

A power law sized distribution reduces skewness of aggregate fluctuations by a constant on the order of 0.5, independent of the exact number of firms. Therefore, the asymptotic behavior of skewness of GDP growth in an economy with a million firms will be a fraction on the order of one half of skewness of GDP growth in our economy with fifteen firms.

We conclude that even though it is hard to infer properties of second moments from our simplified model, this model has strong predictions for skewness and asymmetric behavior which is virtually immune to aggregation and variations in the number of firms.

6.3 Empirical Results

Our model predicts that information processing constraints lead to delays in exit decisions which result in asymmetric aggregate fluctuations. Here we compare these predictions with empirical regularities characterizing the US economy. We use moments such as skewness and correlation which are virtually immune to changes in the number of firms to compare the predictions of our model to regularities in the data.

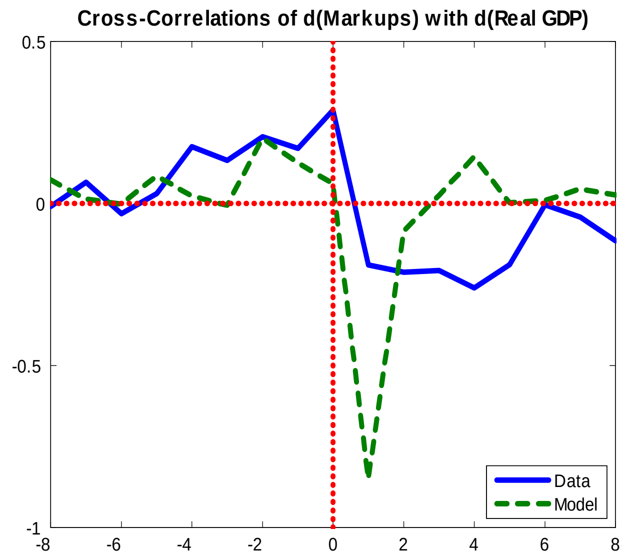

FIGURE 9. **Lead and Lag Properties of Markups.**

Sources: NIPA, Nekarda and Ramey (2010), authors' calculations

	$\rho_{\Delta P,\Delta Y}$	$\rho_{X,\Delta Y}$	$\rho_{R,\Delta Y}$	$\gamma_{\Delta Y}$	$\gamma_{\Delta P}$	γ_X	γ_R
Data	-.24	-.49	.23	-1.07	1.05	1.12	-0.09
Model	-.92	-.65	.34	-4.0	3.8	4.0	1.6

TABLE 10.**Model Performance.**

ρ denotes cross-corellation; γ - skewness; ΔY - growth rate of GDP; ΔP - lagged growth rate of markups; X - exit rate; R - entry rate.

Table 10 shows that our model successfully captures the qualitative properties of business cycles in the U.S. It explains the asymmetry in contractions and expansions and the asymmetry between firm exit and entry rates - facts which served as a motivation of our paper. Another key prediction of our model - the high positive skewness of the growth rate of markups, is also overwhelmingly supported by the data.

We thus provide strong empirical support to a counter-cyclical theory of markups. This theory implies that markups rise sharply in the aftermath of a recession due to a sharp decline in the number of competitors, and then fall gradually in a boom as new businesses populate the economy. This interpretation is consistent with the pattern of cross-correlations of growth rates of GDP and markups depicted in Figure 9. We interpret this picture as follows. When markups are low, GDP falls; after GDP falls, markups rise. These are also the patterns of the data, that our model can explain.

The main feature of US data that our model does not take into account is that it takes more than one quarter for a firm to exit and then for the survivors to realize that many of their competitors are gone, so that they can start charging higher prices or paying lower wages to increase their profit margins. Figure 9 indicates that in the US economy this process takes on average 4-6 quarters, while our simplified model assumes this could be accomplished in a single quarter. In our view, this difference can account for most of the discrepancy between the predictions of the model and the cyclical properties of the data.

7 Conclusion

This paper presents novel empirical evidence on the relationship between asymmetric variations in the number of market participants, markups and exit decisions of firms. We argue that exit decisions play a pivotal role in explaining asymmetric behavior of profit margins and aggregate activity in the U.S. We propose a theoretical framework based on rational inattention theory of Sims (2006) and show that its predictions are consistent with both classical and novel empirical evidence.

We studies the implications of costly information processing for exit decisions of heterogeneous firms in the presence of an aggregate demand externality. The model economy displays cyclical patterns, which are a consequence of the information constraint only. The model economy alternates between long periods of slow expansion and short periods of sharp contraction. In periods of expansion the economy experiences a net increase in the number of firms which create new products and accumulate jobs. These periods of slow expansion are occasionally interrupted by contraction episodes, when large numbers of firms simultaneously layoff workers and go out of business.

In expansions entry of new firms leads to more intense competition, which implies lower profit margins for everybody, while contractions are followed by increases in profit margins of lucky survivors. Because information processing is costly, firms do not exit the market as smoothly as they enter. Instead large numbers of firms simultaneously decide to layoff workers and exit at the same time.

Information plays a dual role in this economy. First, slow information flow delays firm exit. Second, information on the aggregate price plays the role of a coordination device by inducing correlation between individual perceptions and generating large numbers of simultaneous exits.

Thus, rational inattention helps explain the asymmetry between economic expansions and contractions and provides a potential explanation for recurring cycles of business activity in the U.S. economy. Even in its simplicity, the model is consistent with several stylized facts of business cycles in developed countries, such as counter-cyclical markups, counter-cyclical spikes in layoffs and exit rates and prevalence of relatively large contractions over smaller ones.

The mechanism we uncover might yield starkly different policy implications compared to standard business cycle models. Information about demands for individual products and the way in which it is processed play a central role in our business cycle propagation mechanism. In order to perfectly smooth out the cycle, a central planner would need to acquire and process precise information about the different demands for particular products and then command firm exit in a timely manner. This is hardly feasible.

A second-best policy would be for the central planner to provide the information constrained businesses with easy to process (low-bit) systematic information on the aggregate variables. This way the role of the planner would be to put in place a coordinating mechanism for the firms that can enable them to focus on their own demand and make optimal exit decisions.

A policy of managing the entry of new products and the exit of older products has the potential of smoothing out the cycle at the cost of slowing down long-term economic growth. This intuitive prediction is consistent with multiple central planning experiments, undertaken in different parts of the world. The model has the potential to provide estimates of the effect these business cycle smoothing policies would have on long-term economic growth.

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8 Appendix NOT FOR PUBLICATION

8.1 Substitution between products

Here we describe a generalization of the model to the case of non-perfect substitution. The representative household trades off leisure for consumption, maximizing a standard utility function:

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma} - 1}{1-\gamma} - E_t \right), \tag{25}$$

with respect to the supply of labor, E_t, and a Dixit-Stiglitz consumption aggregator, C_t, which weights differenciated products, q_{it}, by their tastes, v_{it}:

$$C_t = \left(\sum_{i=1}^{K_t} v_{it}^{\frac{1}{\sigma}} q_{it}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{26}$$

The household owns all the firms in the economy. It spends wage and profit income on contemporaneous consumption, maximizing utility subject to a budget constraint:

$$\sum_{i=1}^{K_t} p_{it} q_{it} = w_t E_t + \sum_{i=1}^{K_t} \pi_{it}. \tag{27}$$

Maximization yields the following first-order condition, which determines the demand curve for each good indexed by i:

$$p_{it} = w_t C_t^{-\gamma} \left(\frac{C_t}{q_{it}} \right)^{\frac{1}{\sigma}} v_{it}^{\frac{1}{\sigma}}, \tag{28}$$

which is driven by variations in idiosyncratic tastes v_{it}. Variations in tastes are the only source of uncertainty in the economy.

We define a consumption price index as follows:

$$P_t = \frac{1}{C_t} \sum_{i=1}^{K_t} p_{it} q_{it} = w_t C_t^{-\gamma} \tag{29}$$

The economy is populated by K_t firms, which profit from producing and selling differenciated products q_{it} at price p_{it}. In addition to wages, a firm pays a fixed cost of operating the technology, f:

$$\pi_{it} = p_{it}q_{it} - w_t l_{it} - f \tag{30}$$

Firms use identical production functions, which are linear in labor inputs, l_{it} :

$$q_{it} = A l_{it} \tag{31}$$

Within each period firms maximize profits (30) with respect to output, $q_{i,t}$, and labor input, $l_{i,t}$, subject to the production function (31) and given the individual demand curve (28). The first order condition of the firm pins down the optimal level of output as a function of the idiosyncratic shock, $v_{i,t}$:

$$\frac{q_{it}}{A} = \frac{\sigma - 1}{\sigma} C_t^{\frac{1}{\sigma} - \gamma} v_{it}^{\frac{1}{\sigma}} q_{it}^{1 - \frac{1}{\sigma}} \tag{32}$$

We substitute output as a function of taste from (32) into (26) to show how tastes determine consumption and prices:

$$P_t = C_t^{-\gamma} = \frac{1}{A} \frac{\sigma}{\sigma - 1} \left(\sum_{i=1}^{K_t} v_{it} \right)^{-\frac{1}{\sigma - 1}} \tag{33}$$

Without loss of generality normalize operating cost, f, to one and let the wage, w_t, be the numeraire. The expression for profits then simplifies to:

$$\pi_{it} = \mu_{t+1} v_{it} - 1. \tag{34}$$

where the markup, μ_{t+1}, is each firm's sufficient statistic, which characterizes the aggregate state of the economy:

$$\mu_{t+1} = A^{\gamma - 1} \frac{(\sigma - 1)^{\gamma - 1}}{\sigma^{\gamma}} \left(\sum_{i=1}^{K_t} v_{i,t} \right)^{\frac{\gamma - \sigma}{\sigma - 1}} \tag{35}$$

This functional form is equivalent to the one presented in the text, except that γ is substituted by $\frac{\gamma - \sigma}{1 - \sigma}$. Depending on the degree of substitution, $\sigma > 0, \sigma \neq 1$, this expression determines the behavior of the economy instead of γ. The rest of the simulation procedure remains intact. In the limit, as $\sigma \to 0$, products become perfect substitutes, so we arrive at the simplified version of the model presented in the text. We work with the simpler version for the purpose of transparency.

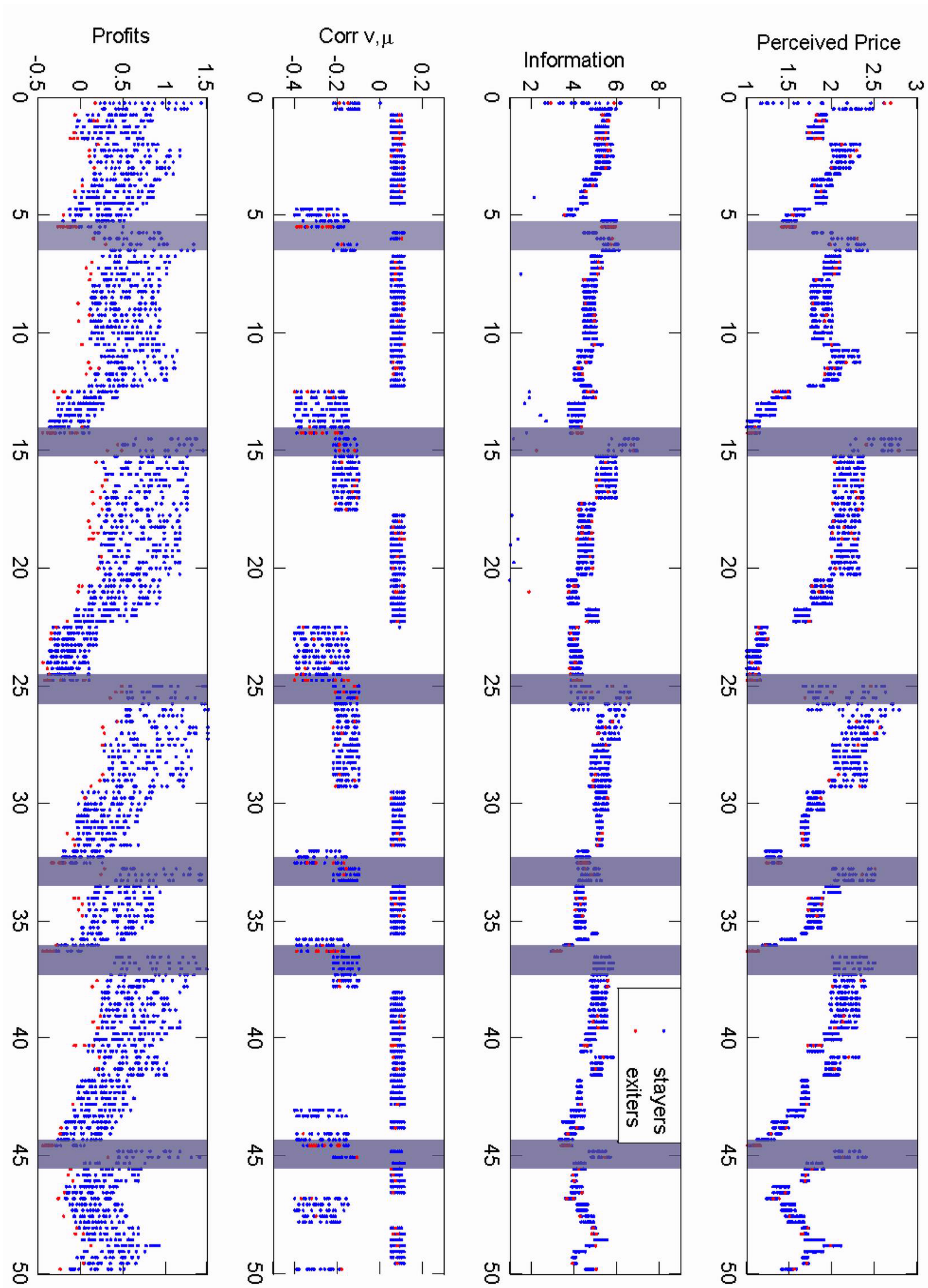

Figure X. **Propagation Mechanism.**

8.2 Bellman Recursion

8.2.1 Concavity of Mutual information in the Belief State.

For a given $p(\pi|s)$, **Mutual Information is concave in** $g(S)$
Proof. Let Z be the binary random variable with $P(Z=0)=\lambda$ and let $S=S_1$ if $Z=0$ and $S=S_2$ if $Z=1$. Let the set of all profits be $\pi_i \in A = \{\pi_1, .., \pi_n\}$Consider

$$\begin{aligned} I(S,Z;\pi) &= I(S;A) + I(Z;A|S) \\ &= I(S;A|Z) + I(Z;A) \end{aligned}$$

Conditional on S, A and Z are independent, $I(A;Z|S)=0$. Thus,

$$\begin{aligned} I(S;A) &\geq I(S;A|Z) \\ &= \lambda(I(S;A|Z=0)) + (1-\lambda)(I(S;A|Z=1)) \\ &= \lambda(I(S_1;A)) + (1-\lambda)(I(S_2;A)) \end{aligned}$$

Q.E.D. ■

Lemma 1 *For a given* $p(\pi|s)$, *the expression (18) is concave in* $g(s)$

8.2.2 The Bellman Recursion is a Contraction Mapping.

Proposition 1. *For the discrete Rational Inattention firm's problem, value recursion* H *and two given functions* V *and* U, *it holds that*

$$||HV - HU|| \leq \beta ||V - U||,$$

with $0 \leq \beta < 1$ *and* $||.||$ *the supreme norm. That is, the value recursion* H *is a contraction mapping.*

Proof. The H mapping displays:

$$HV(g) = \max_p [H^p V(g)]_+,$$

with

$$H^p V(g) = \left[\sum_{s\in S}\left(\sum_{\pi\in A}\pi p(\pi|s)\right) g(s) - \theta\kappa + \beta\sum_{s\in S}\sum_{\pi\in A}(V(g'_\pi(\cdot)))\, p(\pi|s)\, g(s)\right].$$

Suppose that $||HV - HU||$ is the maximum at point g. Let p_1 denote the optimal control for HV under g and p_2 the optimal one for HU

$$HV(g) = [H^{p_1}V(g)]_+ ,$$
$$HU(g) = [H^{p_2}U(g)]_+ .$$

$$\Longrightarrow ||HV(g) - HU(g)|| = [H^{p_1}V(g)]_+ - [H^{p_2}U(g)]_+ .$$

Suppose (without loss of generality) that $HV(g) \leq HU(g)$. Since p_1 maximizes HV at g , it follows that

$$[H^{p_2}V(g)]_+ \leq [H^{p_1}V(g)]_+ .$$

Hence,

$$\begin{aligned} ||HV - HU|| &= \\ ||HV(g) - HU(g)|| &= \\ [H^{p_1}V(g)]_+ - [H^{p_2}U(g)]_+ &\leq \\ [H^{p_2}V(g)]_+ - [H^{p_2}U(g)]_+ &\leq \\ \beta \sum_{w\in W}\sum_{a\in A} \left[\left(V^{p_2}\left(g'_a(\cdot)\right)\right) - \left(U^{p_2}\left(g'_a(\cdot)\right)\right) \right] p_2 g(w) &\leq \\ \beta \sum_{w\in W}\sum_{a\in A} \left(||V - U||\right) p_2 g(w) &\leq \\ \beta\, ||V - U|| . \end{aligned}$$

In the derivation above we can open the positive brackets because the profit function is evaluated at the same point p_2. Then either both brackets do not bind, or only the second binds, or both bind, which implies a $\leq$ sign. Otherwise, the whole expression is exactly equal to zero, which also implies a contraction mapping. Recalling that $0 \leq \beta < 1$ completes the proof. ■

8.2.3 The Bellman Recursion is an Isotonic Mapping

Corollary *For the discrete Rational Inattention firm's problem recursion H and two given functions V and U, it holds that $V \leq U \Longrightarrow HV \leq HU$, that is the value recursion H is an isotonic mapping.*

Proof. Let p_1 denote the optimal control for HV under g and p_2 the optimal one for HU

$$HV(g) = H^{p_1}V(g),$$
$$HU(g) = H^{p_2}U(g).$$

By definition,

$$H^{p_1}U(g) \leq H^{p_2}U(g).$$

From a given g, it is possible to compute $\left.g'_\pi(\cdot)\right|_{p_1}$ for an arbitrary c and then the following will hold $V \leq U \Longrightarrow$

$\forall (g(s), \pi),$

$$V\left(\left.g'_\pi(\cdot)\right|_{p_1}\right) \leq U\left(\left.g'_\pi(\cdot)\right|_{p_1}\right) \Longrightarrow$$

$$\sum_{\pi\in A} V\left(\left.g'_\pi(\cdot)\right|_{p_1}\right) \cdot p_1 g \leq \sum_{\pi\in A} U\left(\left.g'_\pi(\cdot)\right|_{p_1}\right) \cdot p_1 g \Longrightarrow$$

$$\sum_{s\in S}\left(\sum_{\pi\in A}\pi g(s) + \beta\sum_{\pi\in A} V\left(\left.g'_\pi(\cdot)\right|_{p_1}\right)\cdot p_1 g\right)$$
$$\leq \sum_{s\in S}\left(\sum_{\pi\in A}\pi g(s) + \beta\sum_{\pi\in A} U\left(\left.g'_\pi(\cdot)\right|_{p_1}\right)\cdot p_1 g\right) \Longrightarrow$$
$$H^{p_1}V(g) \leq H^{p_1}U(g) \Longrightarrow$$
$$\left[H^{p_1}V(g)\right]_+ \leq \left[H^{p_1}U(g)\right]_+ \Longrightarrow$$
$$\left[H^{p_1}V(g)\right]_+ \leq \left[H^{p_2}U(g)\right]_+ \Longrightarrow$$
$$HV(g) \leq HU(g) \Longrightarrow\ HV \leq HU.$$

Note that g was chosen arbitrarily and, from it, $\left.g'_\pi(\cdot)\right|_{p_1}$ completes the argument that the value function is isotone. ■

8.2.4 The Optimal Value Function is Piecewise Linear

Proposition 2. *If the profit function is weakly quasi-convex and if* $\Pr(\pi_j, S_i)$ satisfies (19) and (21)-(23), *then the optimal* $n-$ *step value function* $V_n(g)$ *can be expressed as:*

$$V_n(g) = \max_{\{\alpha_n^i\}_i}\sum_i \alpha_n(S_i)\, g(S_i)$$

where the $\alpha-$ *vectors*, $\alpha : S \to R$, *are* $|S|-$*dimensional hyperplanes.*

Proof. The proof is done via induction. We assume that all the operations are well-defined in their corresponding spaces. Let Γ be the set that contains constraints (19),(21)-(23) .For planning horizon $n = 0$, we have only to take into account the immediate expected rewards and thus:

$$V_0(g) = \max_{p \in \Gamma} \left[\sum_{s \in S} \left(\sum_{\pi \in A} \pi(s) p \right) g(S) \right] \tag{36}$$

and therefore if I define the vectors

$$\left\{ \alpha_0^i(S) \right\}_i \equiv \left(\sum_{\pi \in A} \pi(s) p \right)_{p \in \Gamma} \tag{37}$$

We have the desired

$$V_0(g) = \max_{\left\{ \alpha_0^i(S) \right\}_i} \left\langle \alpha_0^i, g \right\rangle \tag{38}$$

where $\langle ., . \rangle$ denotes the inner product $\langle \alpha_0^i, g \rangle \equiv \sum_{s \in S} \alpha_0^i(s), g(s)$. For the general case, using equations (16)-(17):

$$V_n(g) = \max_{p \in \Gamma} \left[\begin{array}{c} \sum_{s \in S} \left(\sum_{\pi \in A} \pi(s) p(\pi|S) \right) g(S) + \\ +\beta \sum_{s \in S} \sum_{\pi \in A} \left(V_{n-1} \left(g'_\pi(\cdot)_{p\text{ß}} \right) \right) p(\pi|w) g(s) \end{array} \right] \tag{39}$$

by the induction hypothesis

$$V_{n-1}\left(g(\cdot)|_\pi\right) = \max_{\left\{ \alpha_{n-1}^i \right\}_i} \left\langle \alpha_{n-1}^i, g'_\pi(\cdot) \right\rangle \tag{40}$$

Plugging into the above equation (19) and by definition of $\langle ., . \rangle$,

$$V_{n-1}\left(g'_\pi(\cdot)\right) = \max_{\left\{ \alpha_{n-1}^i \right\}_i} \sum_{s' \in S} \alpha_{n-1}^i \left(\sum_{s \in S} \sum_{\pi \in A} T(\cdot; s, \pi) \frac{\Pr(s, \pi)}{\Pr(\pi)} \right) \tag{41}$$

With the above:

$$V_n(g) = \max_{p\in\Gamma}\left[\begin{array}{c}\sum_{s\in S}\left(\sum_{\pi\in A}\pi(s)\,p\right)g(s)+\\ +\beta\max_{\left\{\alpha_{n-1}^i\right\}_i}\ \sum_{s'\in S}\alpha_{n-1}^i(s')\left(\sum_{s\in S}\left(\sum_{\pi\in A}\frac{T(\cdot;s,\pi)}{\Pr(\pi)}\cdot p\right)g(s)\right)\end{array}\right]$$
$$= \max_{p\in\Gamma}\left[\langle\pi\cdot p,\ g(s)\rangle+\beta\sum_{s\in S}\frac{1}{\Pr(\pi)}\max_{\left\{\alpha_{n-1}^i\right\}_i}\left\langle\sum_{s'\in S}\alpha_{n-1}^i(s')\,T(\cdot;s,\pi)\cdot p,\ g\right\rangle\right] \tag{42}$$

At this point, it is possible to define

$$\alpha_{p,\pi}^j(s) = \sum_{s'\in S}\alpha_{n-1}^i(s')\,T(\cdot : s,\pi)\cdot p. \tag{43}$$

Note that these hyperplanes are independent on the prior g for which I am computing V_n. Thus, the value function amounts to

$$V_n(g) = \max_{p\in\Gamma}\left[\langle\pi\cdot p,\ g\rangle+\beta\sum_{\pi\in A}\frac{1}{\Pr(\pi)}\max_{\left\{\alpha_{p,\pi}^j\right\}_j}\left\langle\alpha_{p,\pi}^j, g\right\rangle\right], \tag{44}$$

and define:

$$\alpha_{p,\pi,g} = \arg\max_{\left\{\alpha_{p,\pi}^j\right\}_j}\left\langle\alpha_{p,\pi}^j, g\right\rangle. \tag{45}$$

Note that $\alpha_{p,\pi,g}$ is a subset of $\alpha_{p,\pi}^j$ and using this subset results into

$$V_n(g) = \max_{p\in\Gamma}\left[\langle\pi(s)\cdot p,\ g\rangle+\beta\sum_{\pi\in A}\frac{1}{\Pr(\pi)}\langle\alpha_{p,\pi,g}, g\rangle\right]$$
$$= \max_{p\in\Gamma}\left\langle\pi\cdot+\beta\sum_{\pi\in A}\frac{1}{\Pr(\pi)}\alpha_{p,\pi,g},\ g\right\rangle. \tag{46}$$

Now

$$\left\{\alpha_n^i\right\}_i = \bigcup_{\forall g}\left\{\pi\cdot p+\beta\sum_{\pi\in A}\frac{1}{\Pr(\pi)}\alpha_{p,\pi,g}\right\}_{p\in\Gamma} \tag{47}$$

is a finite set of linear function parametrized in the action set. Note that a maximum of a piecewise linear convex function and a zero (a constant function) is also piecewise linear and convex. ■

8.2.5 .. and Convex (PCWL)

Proposition 3. *Assuming weak quasi-convexity of the profit function and the conditions of Proposition 1, let V_0 be an initial value function that is piecewise linear and convex. Then the i^{th} value function obtained after a finite number of update steps for a rational inattention consumption-saving problem is also finite, piecewise linear and convex (PCWL).*

Proof. The first task is to prove that $\{\alpha_n^i\}_i$ sets are discrete for all n. The proof proceeds via induction. Assuming quasi-convex profit function and since the optimal policy belongs to Γ, it is straightforward to see that through (37), the set of vectors $\{\alpha_0^i\}_i$,

$$\left\{\alpha_0^i\right\}_i \equiv \left(\sum_{s\in S}\left(\sum_{\pi\in A}\left(\pi\left(\mu,v_i\right)\right)p\left(\pi|s\right)\right)g\left(s\right)\right)_{p\in\Gamma}$$

is discrete. For the general case, observe that for discrete controls and assuming $M=\left|\left\{\alpha_{n-1}^j\right\}\right|$, the sets $\left\{\alpha_{p,\pi}^j\right\}$ are discrete, for a given action p and profits π, I can only generate $\alpha_{p,\pi}^j-$vectors. Now, fixing p it is possible to select one of the M $\alpha_{p,\pi}^j-$vectors for each one of the observed consumption π and, thus, $\{\alpha_n^j\}_i$ is a discrete set. The previous proposition, shows the value function to be convex. The *piecewise-linear* component of the properties comes from the fact that $\{\alpha_n^j\}_i$ set is of finite cardinality. It follows that V_n is defined as a finite set of linear functions. ■

Note also, that the profit function $\pi\left(s\right)=\mu v-1$ is strictly quasi-convex in its arguments. Therefore, the existence and uniqueness of the solution of the value iteration problem of the firm follows from the contraction mapping theorem.

8.3 Pseudocode

Let θ be the shadow cost associated with $\kappa_t = I_t(B_t, D_t)$.

- Step 1: Build the transition matrix $T(\cdot; b_t, d_t)$ convoluting the stochastic properties of the random variables (B, D) on an equi-spaced grid.
- Approximate T by a first-order Markov process.
- Step 2: Build the simplex - an equi-spaced grid to approximate each $g(B_t)$- a simplex point.
- Step 3: For each simplex point, define $p(b_t, d_t)$and initialize $V\left(g'_{\pi_j}(\cdot)\right) = 0$.
- Step 4: For each simplex point, find $p^*(b, d)$ which solves

$$V_0(g(b_t))|_{p^*(b_t,d_t)} = \max_{p(b_t,sd)} \left\{ \sum_{b_t \in \Omega_w} \sum_{d_t \in \Omega_c} (\pi_t(b, d)) \, p^*(b_t, d_t) - \theta \left[I_t(B_t, D_t) \right] \right\}.$$

- Step 5: For each simplex point, compute $g'_{\pi_j}(\cdot) = \sum_{b_t \in \Omega_b} T(\cdot; b_t, d_t) \, p^*(b_t | d_t)$. Use a kernel regression to interpolate $V_0(g(b_t))$ into $g'_{\pi_j}(\cdot)$.
- Step 6: Optimize using csminwel and iterate on the value function to convergence.
- Step 7. For each model, draw from the ergodic $p^*(b, d)$, samples of (b_t, d_t) and use the consumer's F.O.C. to simulate the time series of consumption, prices, markups and expected markups, profits and exit decisions.
- Step 8. Compute the model-simulated empirical distribution of consumption, prices, markups and the idiosyncratic shocks to generate the empirical transition matrix and go back to Step 1.
- Step 9. Iterate until convergence.

Observations

1. Firms' value function takes about 20 iterations to converge.

2. Global equilibrium (law of motion) takes up to 7 iterations to converge.

8.4 Dependence on Variations in Parameters

Here we explore the properties of the full information model and its dependence on changes in various parameters. The behavior of the full-information version is relatively robust to most of the parameter choices. Below are tables for K=15:

	σ_C	σ_P	ρ_C	$\tau_{5-10\%}$	$\tau_{10-20\%}$	$\tau_{>20\%}$	$\gamma_{\Delta C}$	$\gamma_{\Delta P}$	γ_X	γ_R
K=15	0.13	0.13	0.57	1.8	2.6	15	-.18	.26	1.6	1.1
$\gamma = 0.5$	0.23	0.10	0.83	1.7	4.3	20.8	-.47	.53	1.9	1.1
$\gamma = 1.5$	0.11	0.16	0.51	1.9	2.3	15.6	-.24	.23	1.5	1.2
$\gamma = 2.5$	0.10	0.27	0.39	2.1	2.5	8.9	-.35	.44	2.0	1.4
$\varphi = 0.5$	0.12	0.12	0.62	1.8	2.5	20.8	.03	.19	2.0	1.0
$\varphi = 0.2$	0.13	0.13	0.67	2.2	2.8	7.8	-.42	.58	2.3	1.0
$s = 0.025$	0.12	0.13	0.65	2.2	5.1	62.5	-.39	.29	2.6	1.9

K=35:

	σ_C	σ_P	ρ_C	$\tau_{5-10\%}$	$\tau_{10-20\%}$	$\tau_{>20\%}$	$\gamma_{\Delta C}$	$\gamma_{\Delta P}$	γ_X	γ_R
K=35	0.09	0.09	0.59	2.9	8.1	∞	-.13	.15	1.1	0.7
$\gamma = 0.5$	0.19	0.08	0.81	3.6	31.5	∞	-.14	.16	0.5	0.7
$\gamma = 1.5$	0.08	0.11	0.60	2.3	15.6	∞	-.01	.08	0.8	0.7
$\gamma = 2.5$	0.06	0.16	0.35	2.3	5.4	∞	-.02	.28	1.1	0.7
$\varphi = 0.5$	0.08	0.08	0.69	2.6	10.4	∞	-.14	.16	1.0	0.7
$\varphi = 0.2$	0.08	0.08	0.74	3.2	10.4	∞	-.06	.02	1.1	0.7

and K=70:

	σ_C	σ_P	ρ_C	$\tau_{5-10\%}$	$\tau_{10-20\%}$	$\tau_{>20\%}$	$\gamma_{\Delta C}$	$\gamma_{\Delta P}$	γ_X	γ_R
K=70	0.07	0.07	0.64	5.3	∞	∞	-.05	.08	0.5	0.6
$\gamma = 0.5$	0.10	0.05	0.81	5.1	∞	∞	-.06	.06	1.5	0.6
$\gamma = 1.5$	0.05	0.07	0.32	2.8	62.5	∞	-.14	.17	1.3	0.6
$\varphi = 0.2$	0.09	0.09	0.87	4.1	∞	∞	-.09	.13	0.8	0.6

8.5 Properties of Skewness

Note that skewness of a sum of independent random variables can be expressed as follows:

$$\gamma_{\Sigma X_i} = \frac{E\left[(\Sigma X_i - EX_i)^3\right]}{\left(\sigma_{\Sigma X_i}\right)^3} = \frac{\Sigma E(X_i - EX_i)^3}{\left(\sigma_{\Sigma X_i}\right)^3} = \frac{\Sigma_i\left[(VarX_i)^{\frac{3}{2}} SkewX_i\right]}{(\Sigma_i VarX_i)^{\frac{3}{2}}}$$

Let output in the economy be a sum of i.i.d. outputs of individual firms: $Y_t = \Sigma S_i$. Then, aggregate GDP growth follows

$$\Delta y = \frac{\Delta Y}{Y} = \Sigma \frac{S_i}{Y} \sigma_i \varepsilon_{it} = \Sigma s_i \varepsilon_{it}$$

Let for simplicity firms have the same volatility σ and skewness γ, then skewness of growth is:

$$\gamma_{\Delta y} = \gamma_{\Sigma s_i \varepsilon_{it}} = \frac{\Sigma_i\left[(VarX_i)^{\frac{3}{2}} SkewX_i\right]}{(\Sigma_i VarX_i)^{\frac{3}{2}}} = \frac{\Sigma_i\left[\left(s_i^2\sigma^2\right)^{\frac{3}{2}} Skew(s_i\varepsilon_i)\right]}{\left(\sigma^2\Sigma_i s_i^2\right)^{\frac{3}{2}}} = \frac{\Sigma_i s_i^3 \gamma_i}{\left(\Sigma_i s_i^2\right)^{\frac{3}{2}}}$$

Using the properties of a power distribution:

$$\Sigma_i s_i^2 \sim \frac{1}{N^2}\Sigma\left(\frac{i}{N}\right)^{-\frac{2}{\zeta}} = N^{\frac{2}{\zeta}-2}\left(\Sigma i^{-\frac{2}{\zeta}}\right)$$
$$\Sigma_i s_i^3 \sim \frac{1}{N^3}\Sigma\left(\frac{i}{N}\right)^{-\frac{3}{\zeta}} = N^{\frac{3}{\zeta}-3}\left(\Sigma i^{-\frac{3}{\zeta}}\right)$$

Hence, we can express the skewness of GDP as the product of skewness of processes for individual firms multiplied by a ratio of finite sums:

$$\gamma_{\Delta y} = \gamma \frac{\Sigma_i s_i^3}{\left(\Sigma_i s_i^2\right)^{\frac{3}{2}}} \sim \gamma \frac{\left(\frac{1}{N}\right)^3 N^{\frac{3}{\zeta}}}{\left(\frac{1}{N}\right)^3 N^{\frac{3}{\zeta}}} \frac{\left(\Sigma i^{-\frac{3}{\zeta}}\right)^{\frac{1}{2}}}{\left(\Sigma i^{-\frac{2}{\zeta}}\right)^{\frac{3}{2}}} = \gamma\left(\frac{\Sigma i^{-\frac{3}{\zeta}}}{\left(\Sigma i^{-\frac{2}{\zeta}}\right)^3}\right)^{\frac{1}{2}}$$

If Zipf's law holds, $\zeta = 1$ (Pareto distribution), then $\gamma_{\Delta y} \sim 0.52\gamma$. If we adopt the standard estimate for the US economy, $\zeta = 1.055$, the result is little changed: $\gamma_{\Delta y} \sim 0.48\gamma$. However, if we assume the other extreme $\zeta = 2$ (no fat tails), then $\gamma_{\Delta y} \sim 0.06\gamma$.

The key message from this derivation is that the number of firms has no effect on the asymptotic skewness of GDP growth. A power law sized distribution reduces skewness of aggregate fluctuations by a constant, independent of the exact number of firms. A similar argument can be made for cross-correlations.