A Model of the Small Open Economy with a Financial Accelerator

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Cespedes, Chang, Velasco 2000

Description of the model

Firms:

Constant returns to scale, flexible or sticky wages, no profits.

(0) $L_t = 1 \quad or \quad E_t L_{t+1}^v = 1$

(1) $Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}$

(2) $R_t K_{t-1} = \alpha P_t Y_t$

(3) $W_t L_t = (1-\alpha) P_t Y_t$

Consumers

CRS aggregator of home and foreign goods, all labor income spent on consumption.

(4) $\gamma^{\gamma} (1-\gamma)^{1-\gamma} C_t = C_{Ht}^{\gamma} C_{Ft}^{1-\gamma}$

(5) $Q_t = P_t^{\gamma} S_t^{1-\gamma}$

(6) $P_t C_{Ht} + S_t C_{Ft} = W_t L_t$

(7) $\frac{1-\gamma}{\gamma} = \frac{S_t C_{Ft}}{P_t C_{Ht}}$

Capitalists

Financial accelerator, investment financed through debt, full depreciation, accumulation of net worth:

(8) $P_t N_t + S_t D_t = Q_t K_t$

(9) $(1+\eta_t) = \chi \left(\frac{Q_t K_t}{P_t N_t} \right) = \left(\frac{Q_t K_t}{P_t N_t} \right)^{\mu}$

(10) $E_t \left[\frac{\alpha P_{t+1} Y_{t+1}}{Q_t K_t} \right] = E_t \left[\frac{S_{t+1}}{S_t} \right] (1+\rho_t)(1+\eta_t)$

(11) $P_t N_t = \delta \left[R_t K_{t-1} \exp(\xi_t) - (1+\rho_{t-1})(1+\eta_{t-1}) S_t D_{t-1} \right]$

Equilibrium conditions

Resource constraints and persistent shocks to technology, export, world interest rate and financial conditions:

(12) $P_t Y_t = \gamma Q_t (C_t + K_t) + S_t X_t$

(13) $Q_t C_t = W_t L_t = (1-\alpha) P_t Y_t$

(14) $A_t = A_0^{1-\rho_A} A_{t-1}^{\rho_A} \exp(\varepsilon_{At})$

(15) $\rho_t = (1-\rho_i) \rho_0 + \rho_i \rho_{t-1} + \varepsilon_{it}$

(16) $X_t = X_0^{1-\rho_A} X_{t-1}^{\rho_A} \exp(\varepsilon_{Xt})$

(17) $\xi_t = \rho_{\xi} \xi_{t-1} + \varepsilon_{\xi t}$

State: $\{Y_t, K_t, C_t, C_H, C_F, N_t, D_t, L_t, P_t, S_t, Q_t, W_t, R_t, \eta_t, \rho_t, A_t, X_t, \xi_t\}$

Stationary version

Here all prices are in real terms and P_t is normalized to 1.

(0) $L_t = 1 \quad or \quad E_t L_{t+1}^{v} = 1$

(1) $Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}$

(2) $r_t K_{t-1} = \alpha Y_t$

(3) $w_t L_t = (1-\alpha) Y_t$

(5) $q_t = s_t^{1-\gamma}$

(6) $C_{Ht} + s_t C_{Ft} = w_t L_t$

(7) $\frac{1-\gamma}{\gamma} = \frac{s_t C_{Ft}}{C_{Ht}}$

(8) $N_t + s_t D_t = q_t K_t$

(9) $(1+\eta_t) = \chi\left(\frac{q_t K_t}{N_t}\right) = \left(\frac{q_t K_t}{N_t}\right)^{\mu}$

(10) $E_t\left[\frac{\alpha Y_{t+1}}{q_t K_t}\right] = E_t\left[\frac{s_{t+1}}{s_t}\right](1+\rho_t)(1+\eta_t)$

(11) $N_t = \delta\left[r_t K_{t-1}\exp(\xi_t) - (1+\rho_{t-1})(1+\eta_{t-1}) s_t D_{t-1}\right]$

(12) $Y_t = \gamma q_t (C_t + K_t) + s_t X_t$

(13) $q_t C_t = w_t L_t = (1-\alpha) Y_t$

(14) $A_t = A_0^{1-\rho_A} A_{t-1}^{\rho_A} \exp(\varepsilon_{At})$

(15) $\rho_t = (1-\rho_i)\rho_0 + \rho_i \rho_{t-1} + \varepsilon_{it}$

(16) $X_t = X_0^{1-\rho_A} X_{t-1}^{\rho_A} \exp(\varepsilon_{Xt})$

(17) $\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_{\xi t}$

State: $\left\{ \begin{array}{ll} Y_t, K_t, C_t, C_H, C_F, N_t, D_t, L_t & 8 \\ s_t, q_t, w_t, r_t, \eta_t, \rho_t, A_t, X_t, \xi_t & 9 \end{array} \right\}$

Parameters: $\left\{ \begin{array}{ll} \alpha, \gamma, v, \eta, lev, \mu, \delta, A_0, X_0, \rho_0 & 8+2 \\ \rho_A, \rho_X, \rho_I, \rho_\xi, \sigma_A, \sigma_X, \sigma_I, \sigma_\xi & 8 \end{array} \right\}$

So that: $\mu = -\log(1+\eta)/\log(1-1/lev), \qquad 1 = \delta(1+\rho)(1+\eta)$

α	γ	ρ_0	ρ_A	ρ_X	ρ_I	ρ_ξ	Y_0
0.35	0.6	0.04	0.8	0.8	0.5	0.5	0.5

case	η	lev	μ	δ	A_0	X_0
robust	0.02	6	0.11	0.94	0.55	9.385
vulnerable	0.04	1.2	0.02	0.92	0.54	11.294
shitty	0.12	1.001	0.016	0.86	0.52	19.229

Finding the steady-state:

$PN + SD = QK \qquad (1+\eta) = \left(\frac{QK}{PN}\right)^{\mu} \qquad \frac{QK}{PN} = \frac{1}{1-1/lev} = (1+\eta)^{1/\mu}$

$\mu = -\log(1+\eta)/\log(1-1/lev) \qquad \frac{QK}{SD} = lev \qquad \frac{PN}{QK} + \frac{SD}{QK} = 1 \qquad RK = \alpha PY$

$\frac{\alpha PY}{QK} = (1+\rho)(1+\eta) \qquad \frac{PN}{QK} = \delta\left[\frac{\alpha PY}{QK} - (1+\rho)(1+\eta)\frac{SD}{QK}\right] \qquad 1 = \delta(1+\rho)(1+\eta)$

$A = A_0 \qquad \rho = \rho_0 \qquad X = X_0 \qquad \xi = 0 \qquad P = 1$

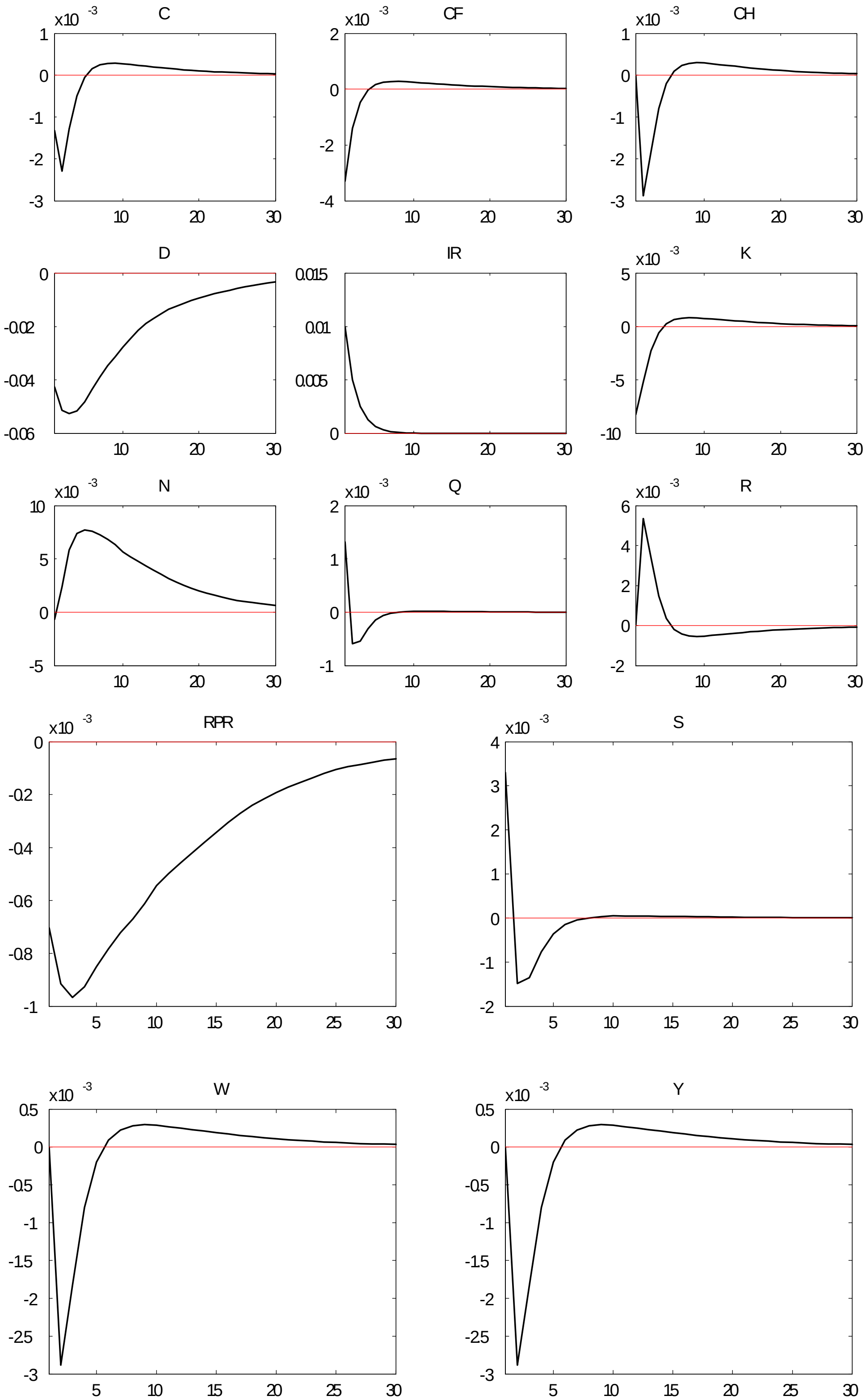
$Y = AK^{\alpha} \qquad QC = W = (1-\alpha) PY \qquad Y = \gamma(1-\alpha) Y + \gamma(1+\rho)(1+\eta)\alpha Y + SX$

$Q = P^{\gamma} S^{1-\gamma} = S^{1-\gamma} \qquad S = \left(\frac{\alpha A K^{\alpha-1}}{(1+\rho)(1+\eta)}\right)^{\frac{1}{1-\gamma}}$

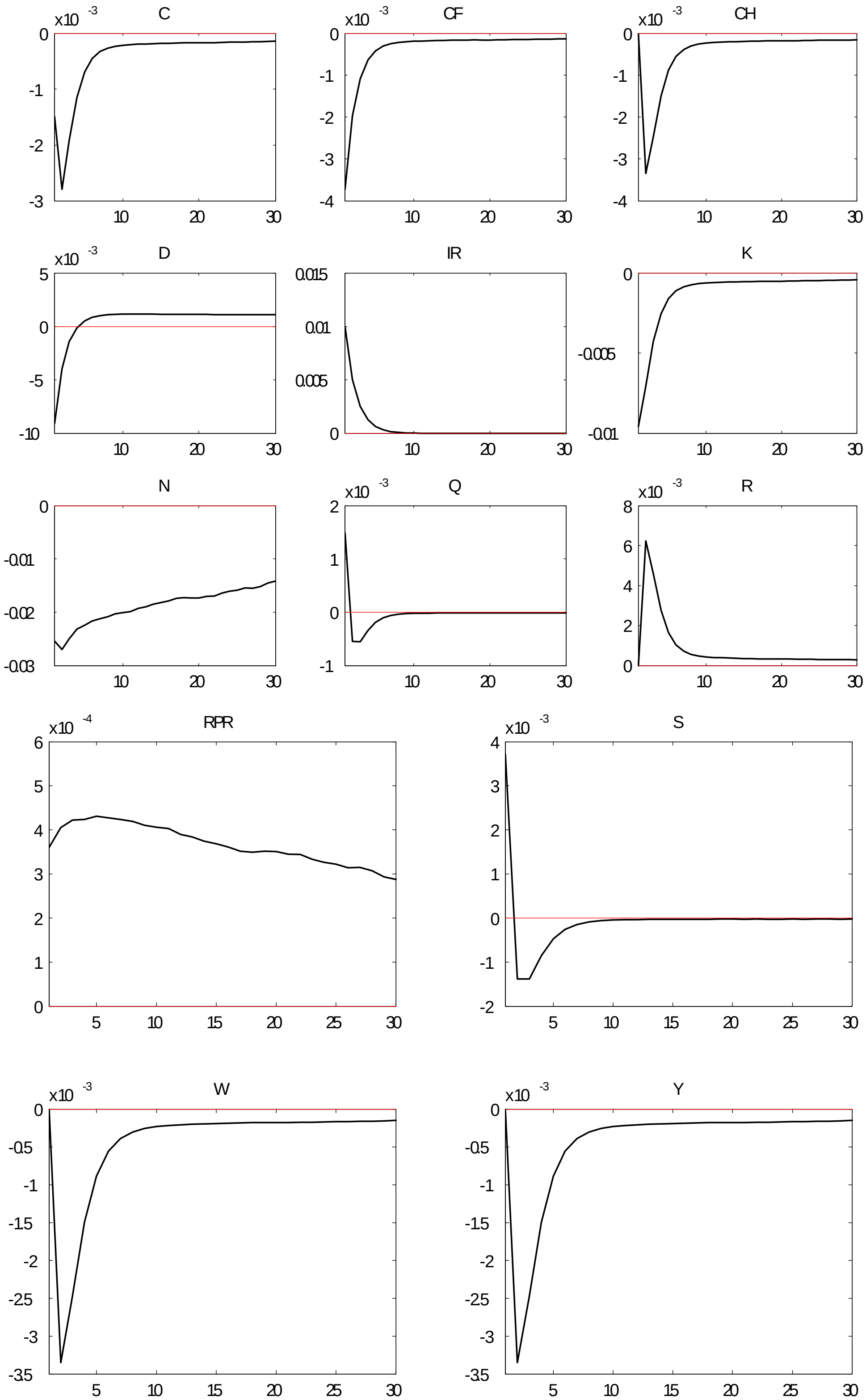
$$\boxed{(1-\gamma(1-\alpha) - \gamma(1+\rho)(1+\eta)\alpha) = \left(\frac{\alpha}{(1+\rho)(1+\eta)}\frac{1}{K}\right)^{\frac{1}{1-\gamma}} X}$$

$\left(\frac{\alpha A K^{\alpha-1}}{(1+\rho)(1+\eta)}\right)^{\frac{1}{1-\gamma}} C_F = (1-\gamma) W = (1-\gamma)(1-\alpha) A K^{\alpha} \qquad C_H = \gamma(1-\alpha) Y$

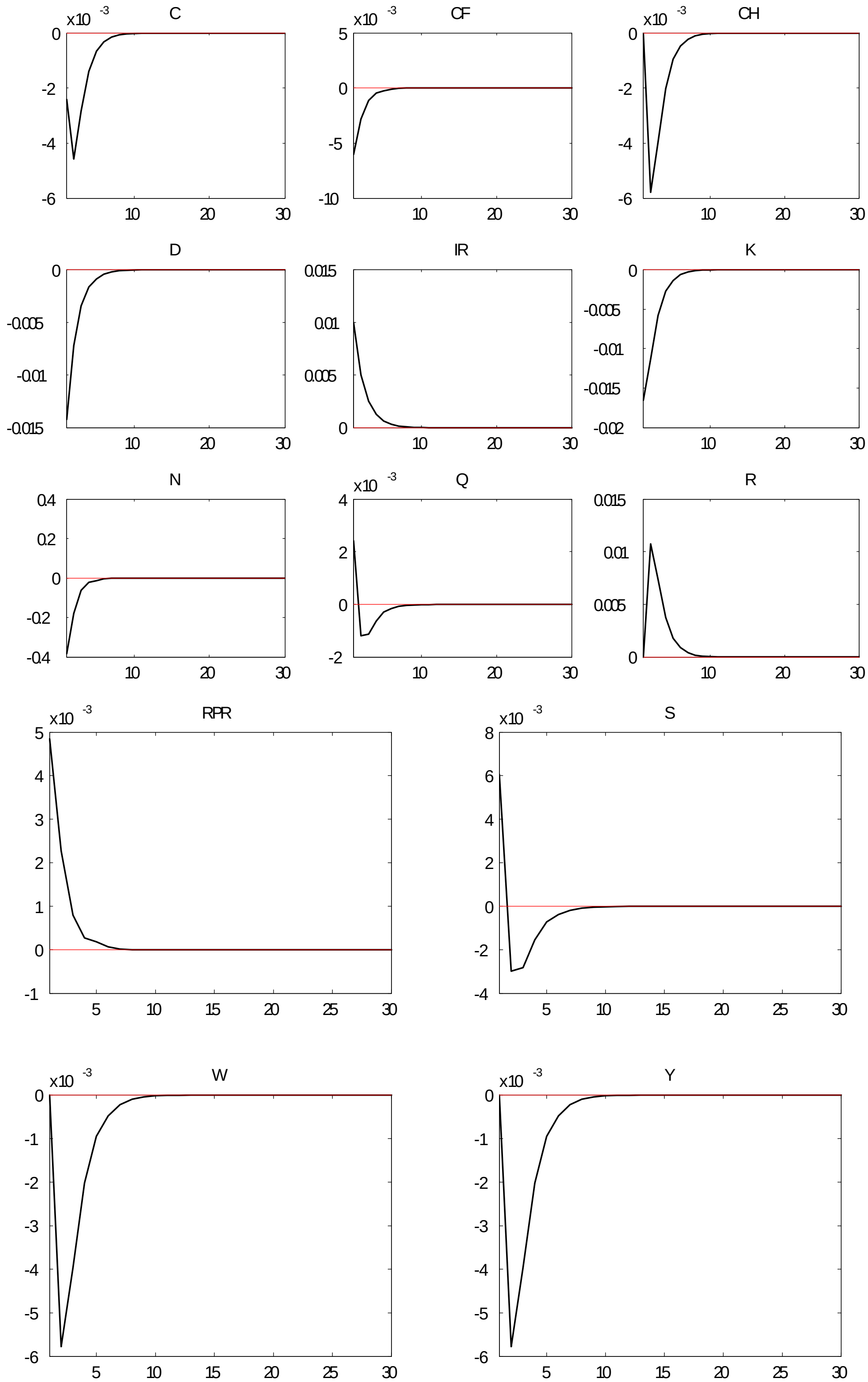
Impulse responses to a positive World Interest Rate shock (robust):



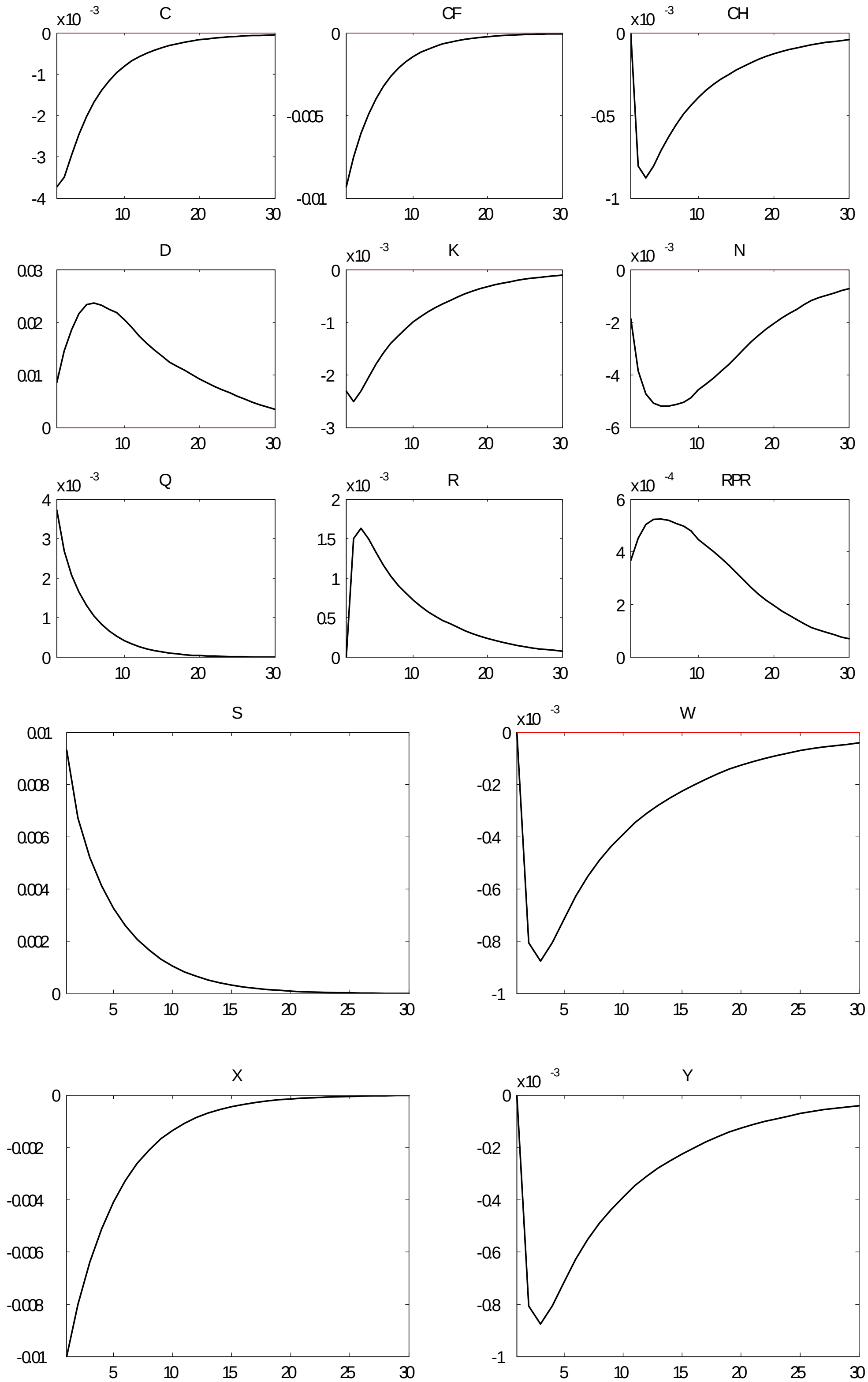
Impulse responses to a positive World Interest Rate shock (vulnerable):



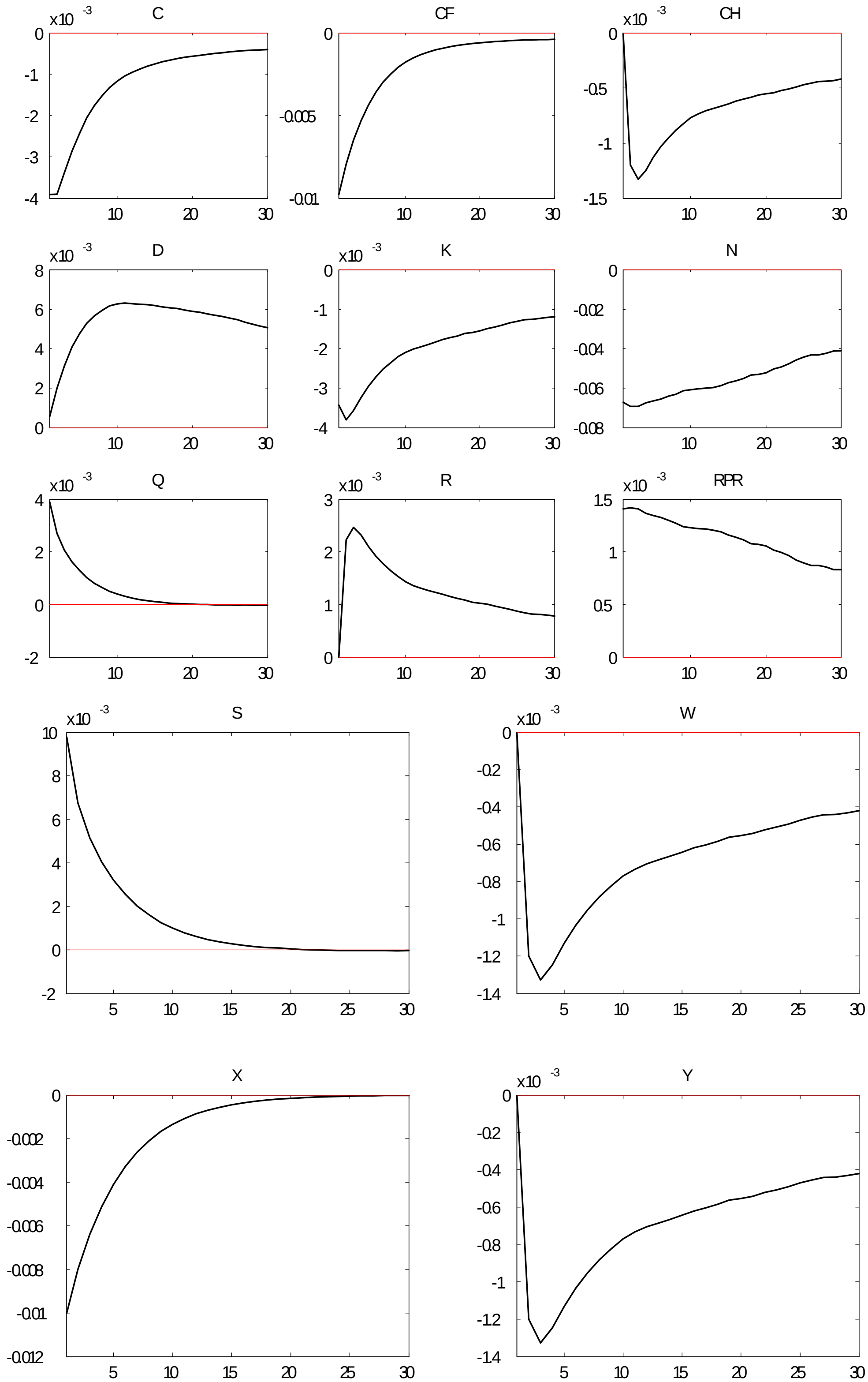
Impulse responses to a positive World Interest Rate shock (shitty):



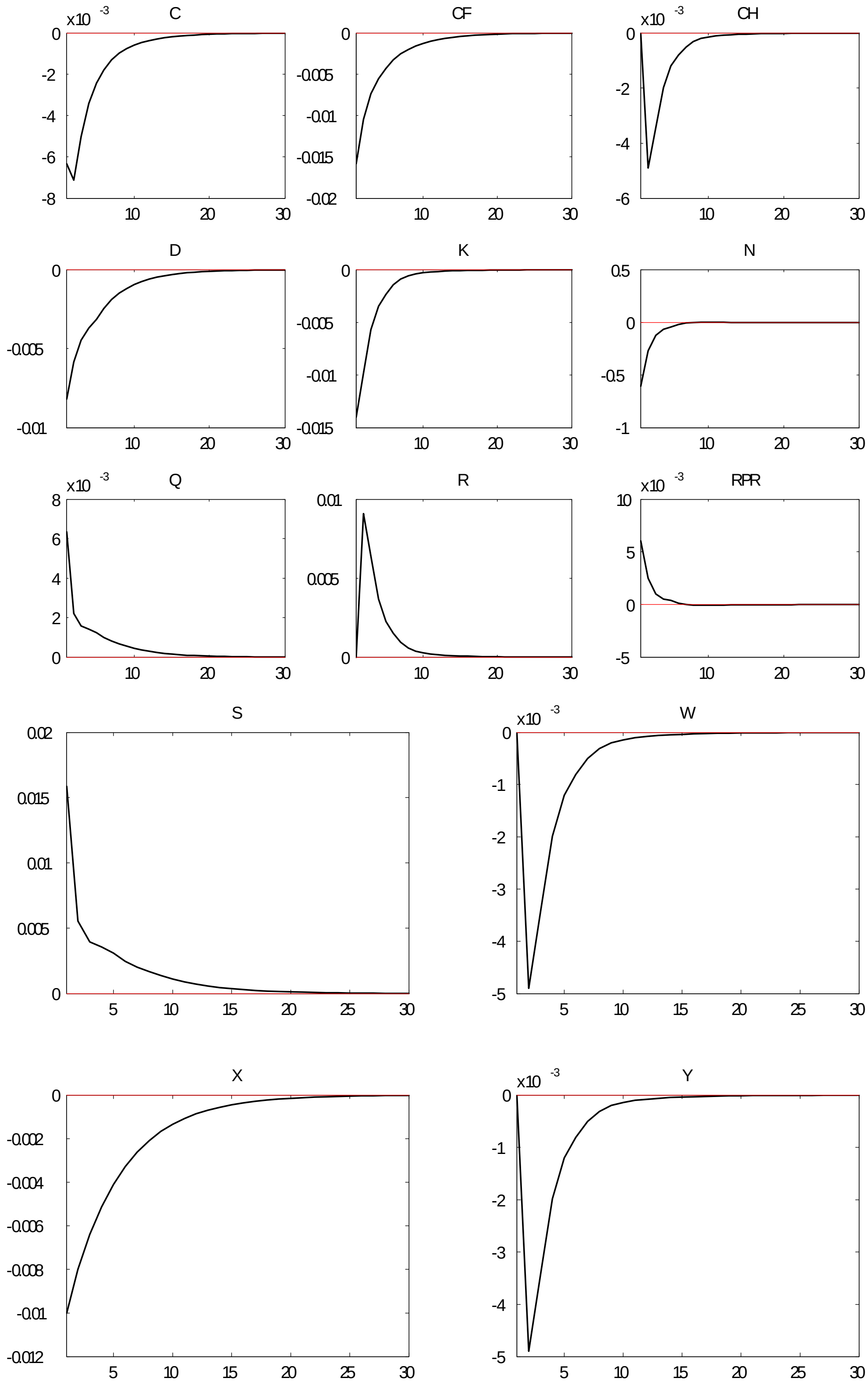
Impulse responses to a negative Export Shock (robust):



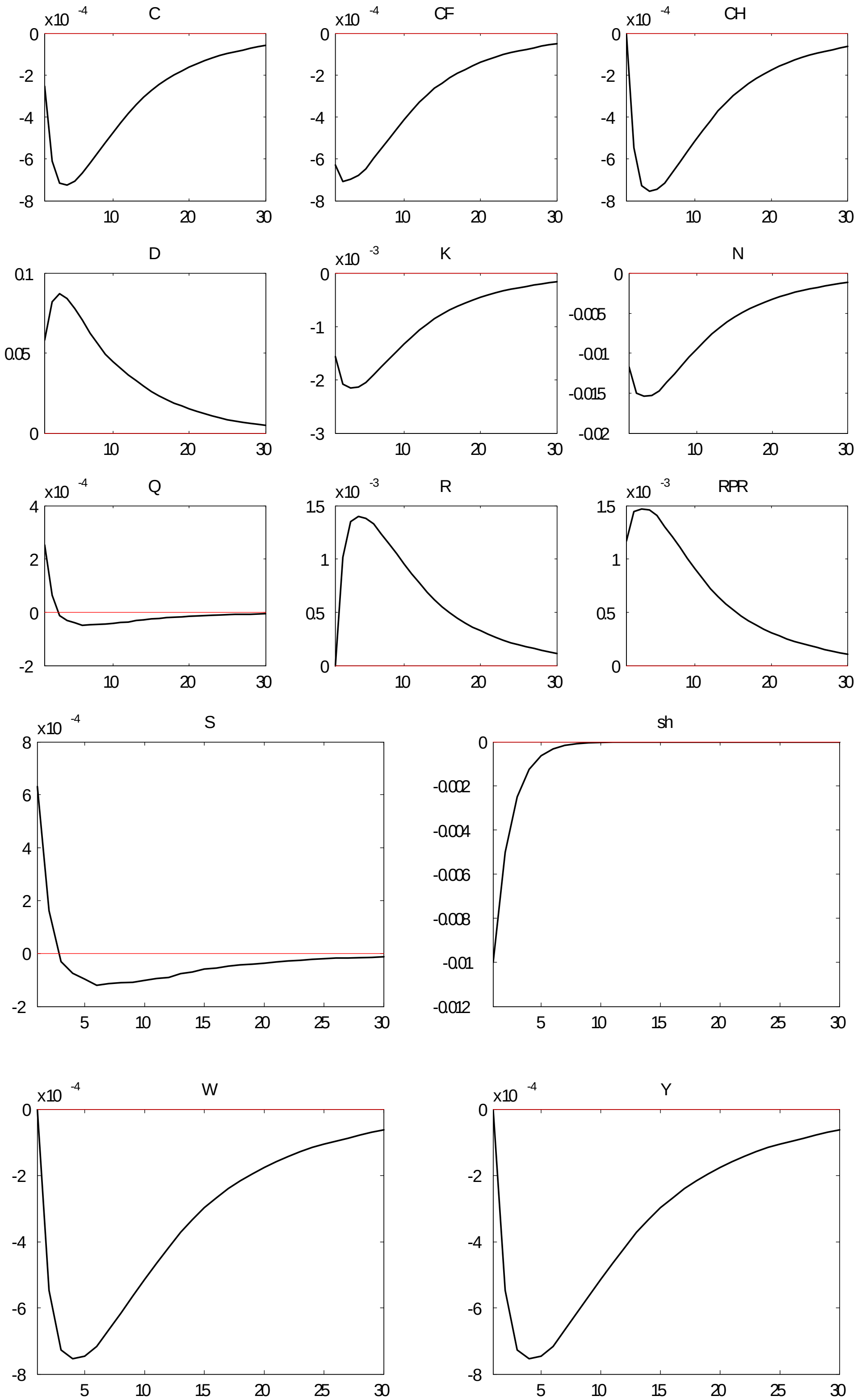
Impulse responses to a negative Export Shock (vulnerable):



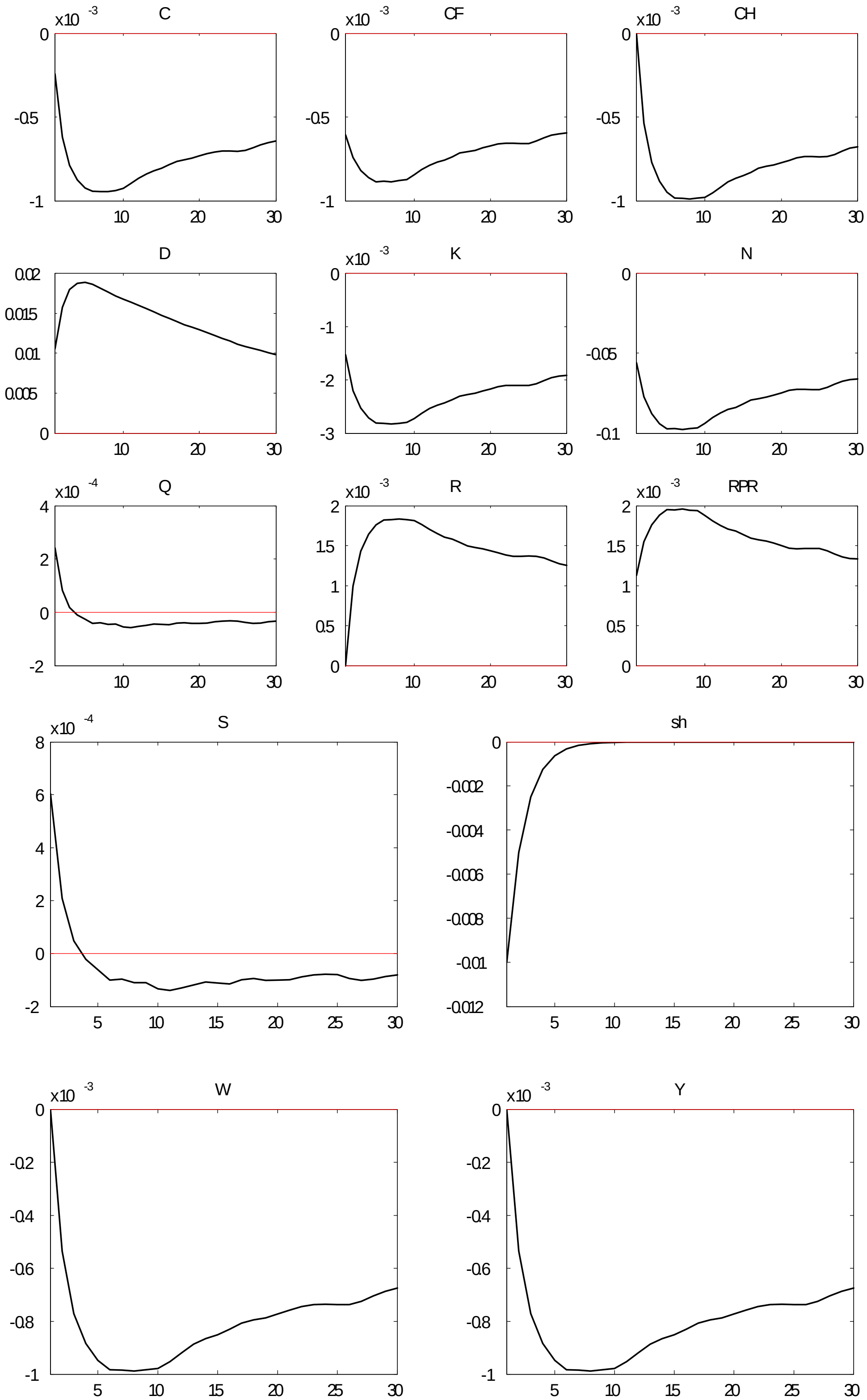
Impulse responses to a negative Export Shock (shitty):



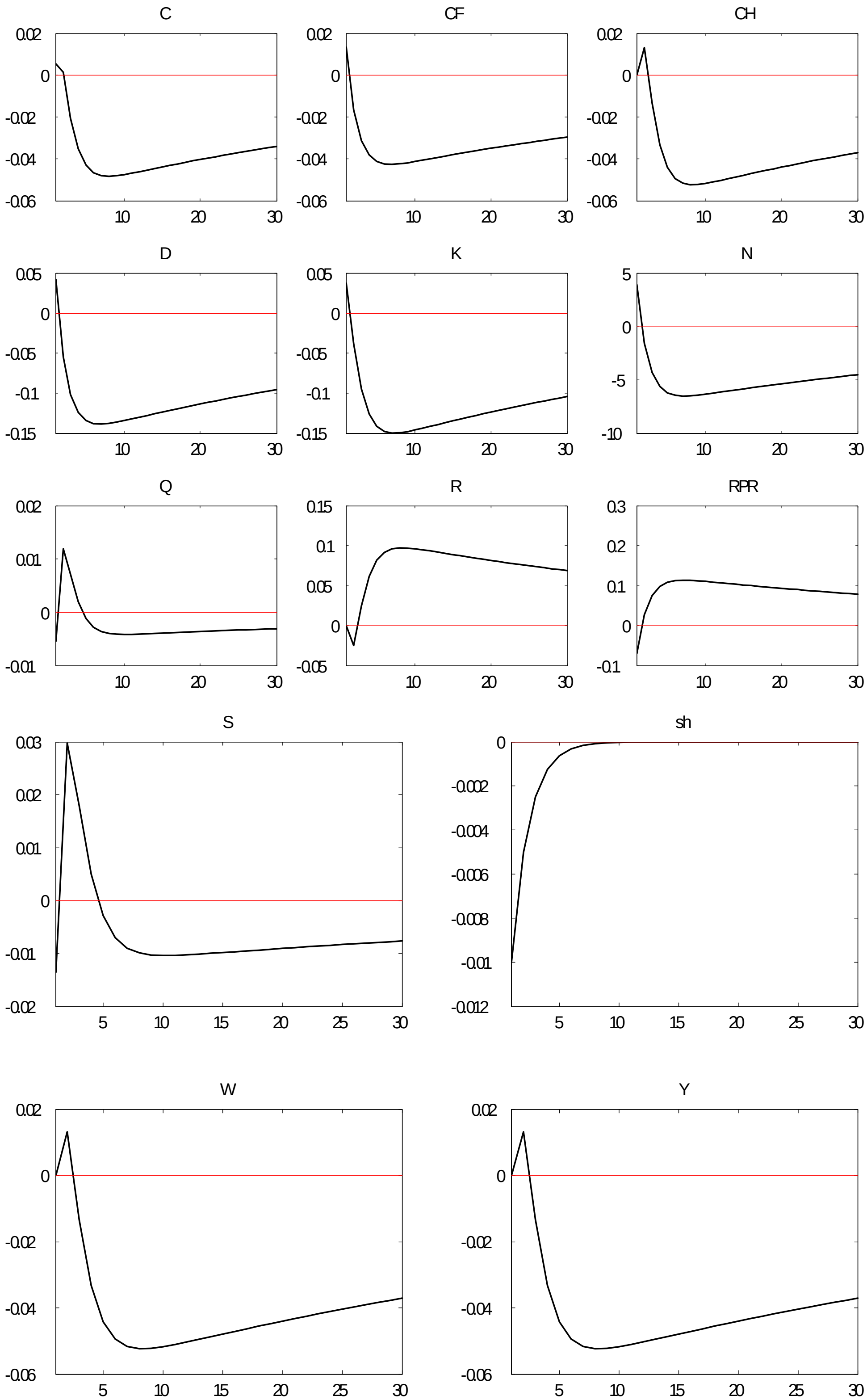
Impulse responses to a negative Financial shock (robust):



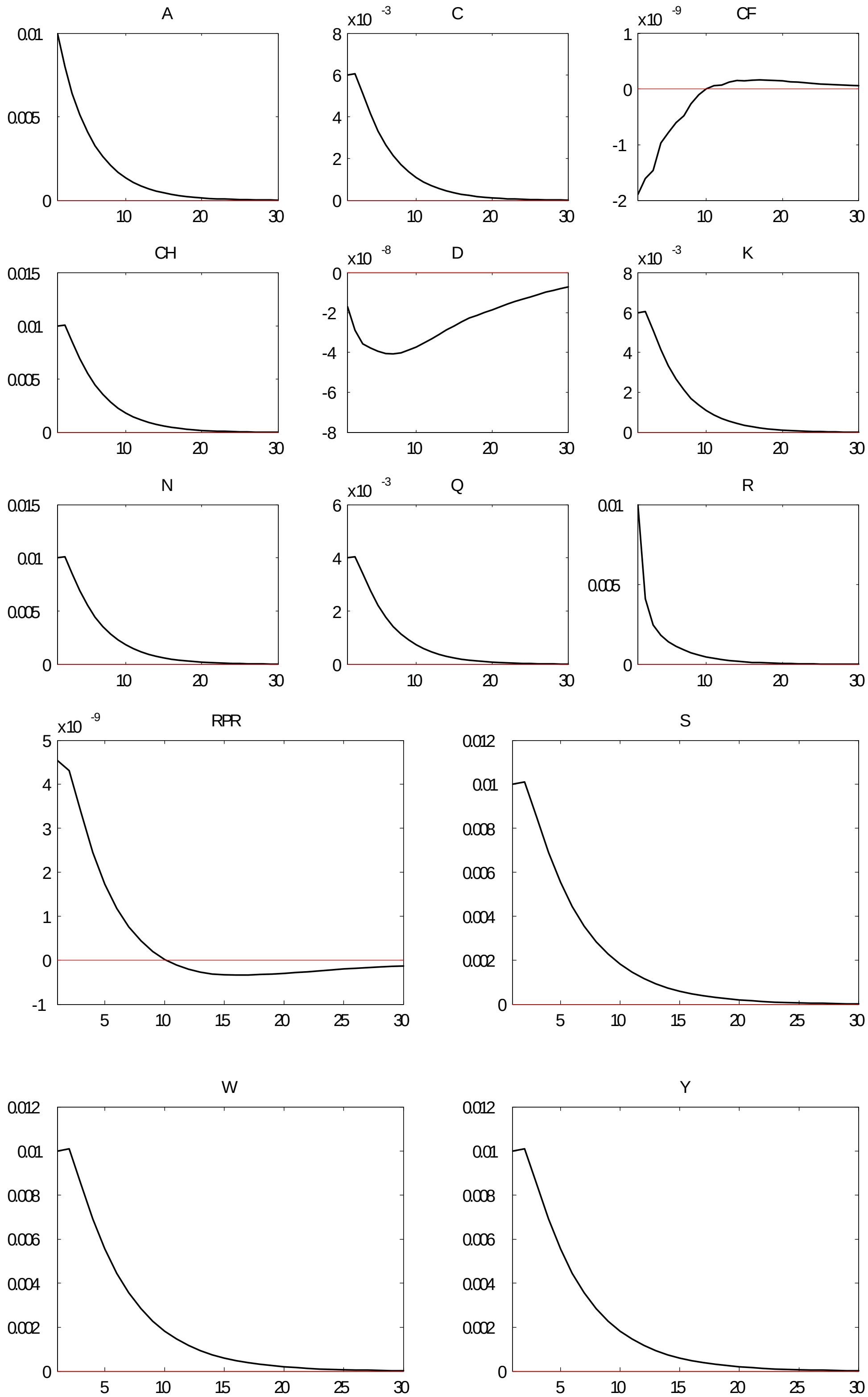
Impulse responses to a negative Financial shock (vulnerable):



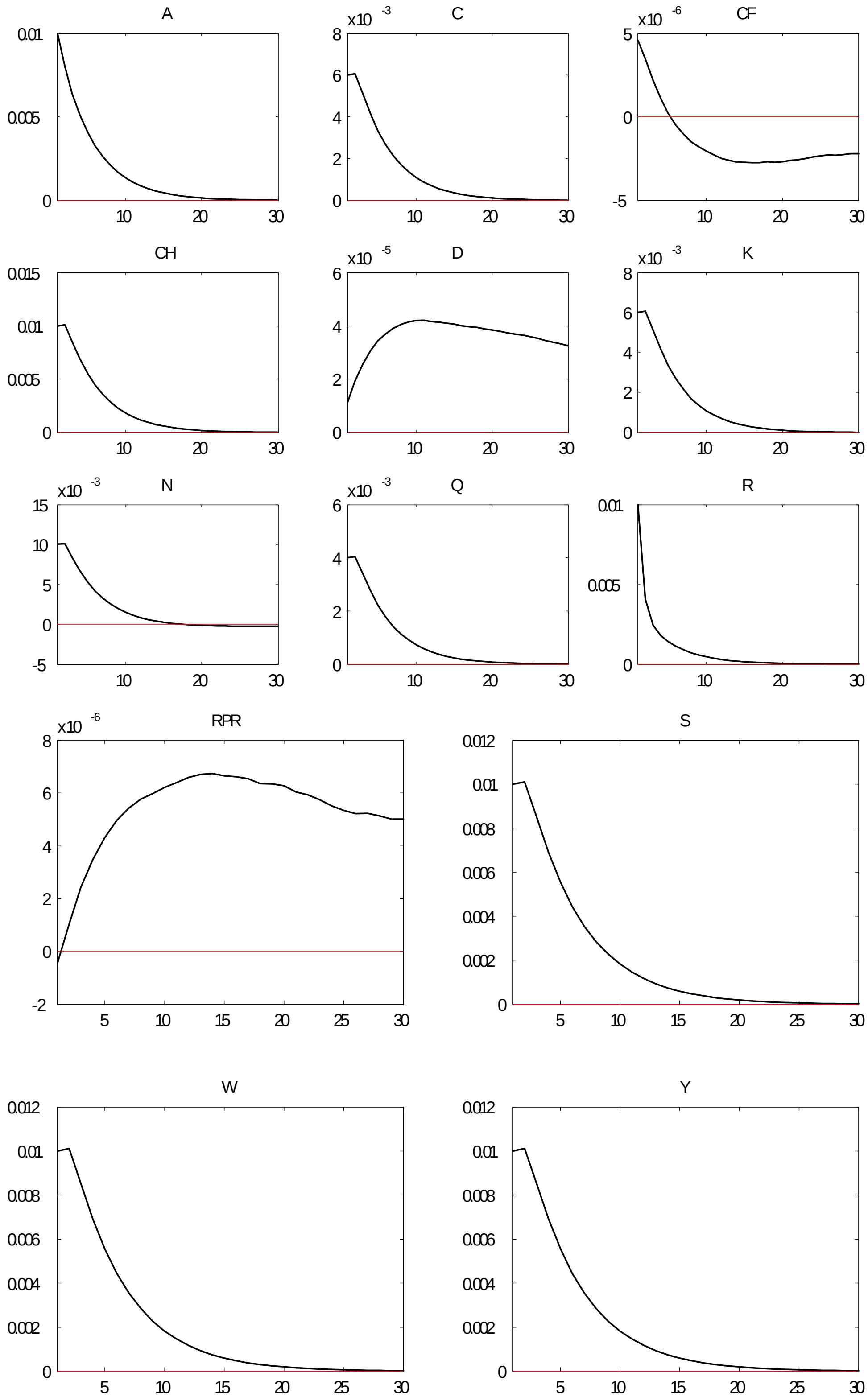
Impulse responses to a negative Financial shock (shitty, first order):



Impulse responses to a positive TFP shock (robust):



Impulse responses to a positive TFP shock (vulnerable):



Impulse responses to a positive TFP shock (shitty):