Midterm Answer Key

Prepared by Alberto Naudon

November 16, 2005

1 Profit maximization

(a)

Let $p^\lambda = \lambda p^0 + (1-\lambda)\, p^1$ be a convex combination ($\lambda \in (0,1)$) of p^0 and p^1 and y^λ the optimum choice when prices are p^λ. Let $\Pi\left(p^0\right) = p^0 \cdot y^0$ and $\Pi\left(p^1\right) = p^1 \cdot y^1$ be the maximized profit when prices are p^0 and p^1. Then $\lambda \Pi\left(p^0\right) = \lambda p^0 \cdot y^0 \geq \lambda p^0 \cdot y^\lambda$ and $(1-\lambda)\,\Pi\left(p^1\right) = (1-\lambda)\, p^1 \cdot y^1 \geq (1-\lambda)\, p^1 \cdot y^\lambda$ Summing both inequalities, $\lambda \Pi\left(p^0\right) + (1-\lambda)\,\Pi\left(p^1\right) \geq \lambda p^0 \cdot y^\lambda + (1-\lambda)\, p^1 \cdot y^\lambda = \Pi\left(p^\lambda\right)$.

(b)

Production set ($\mathcal{Y}$) should be strictly convex. The prove is by contradiction.

Suppose that y^0 and y^1 with $y^0 \neq y^1$ maximized profits at prices p., then $p \cdot y^0 = p \cdot y^1$. Consider the convex combination $y^\lambda = \lambda y^0 + (1-\lambda)\, y^1$ and notice that $p \cdot y^\lambda = \lambda p \cdot y^0 + (1-\lambda)\, p \cdot y^1 = p \cdot y^0 = p \cdot y^1$. But if the production set is strictly convex,.$y^\lambda = \lambda y^0 + (1-\lambda)\, y^1 \in \text{int}\mathcal{Y}$, then $p \cdot y^\lambda < p \cdot y^0 = p \cdot y^1$, which is a contradiction.

(c)

The key point is to realized that the production set can be written a constraint (or a set of constraints). Profit function is defined by $\Pi(p) = \max_z \{p \cdot y \mid g(y) \geq 0\}$, the Lagrangian associated to this problem is, $\mathcal{L} : p \cdot y + \lambda g(y)$. By the envelope theorem,

$$\frac{\partial}{\partial y_j}\Pi(p) = \frac{\partial}{\partial y_j}(p \cdot y) = p_j$$

then

$$\frac{\partial^2}{\partial y_j^2}\Pi(p) = \frac{\partial}{\partial y_j}p_j$$

Note that if p_j is the price of an input $\frac{\partial}{\partial y_j}(p \cdot y) = -p_j$ and $\frac{\partial^2}{\partial y_j^2}\Pi(p) = -\frac{\partial}{\partial y_j}p_j$

(d)

From (a) we know that $\Pi(p)$ is convex, then $\frac{\partial^2}{\partial y_j^2}\Pi(p) = \frac{\partial}{\partial y_j}p_j \geq 0$.

(e)

From (a) we know that $\Pi(p)$ is convex, then if p_j is the price of an input $\frac{\partial^2}{\partial y_j^2}\Pi(p) = -\frac{\partial}{\partial y_j}p_j \geq 0$, then $\frac{\partial}{\partial y_j}p_j \leq 0$.

2 Q2

(a)

Two ways to do this exercise.

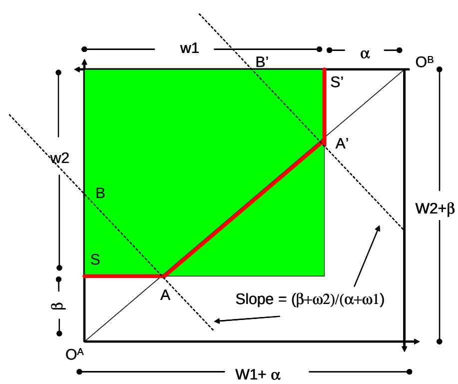
(I) Directly: For any interior point $MRS^A = MRS^B$, in this case that implies

$$\frac{\beta + x_2^A}{x_1^A} = \frac{x_2^B}{\alpha + x_1^B} = \frac{\beta + \omega_2}{\alpha + \omega_1} \tag{1}$$

Where the last equality use the ratio rule. Therefore, the equilibrium prices can be obtained by $MRS^A = MRS^B = \left(\frac{\beta+\omega_2}{\alpha+\omega_1}\right)^{\frac{1}{2}} = \frac{p_1}{p_2}$, which is a constant.

(II) Indirectly: let $c_2^A = \beta + x_2^A$ and $c_1^B = \alpha + x_1^B$ The total endowment with the "new" variables is given by $\widetilde{w} = (\alpha + \omega_1, \beta + \omega_2)$. Preferences defines over this "new" variables are identical and homothetic. RA argument implies that for any interior point $\frac{p_1}{p_2} = MRS\left(\widetilde{w}\right) = \left(\frac{\beta+\omega_2}{\alpha+\omega_1}\right)^{\frac{1}{2}}$, which is constant.

Note that using method (II) is easy to draw the Edgeworth box. The feasible area is the green one.



(b)

The range of possible prices are given by the solution at the corners. When A is on the boundary prices should be equal to $MRS^B = \left(\frac{\alpha+x_1^B}{x_2^B}\right)^{\frac{1}{2}} \left(= \left(\frac{c_1^B}{x_2^B}\right)^{\frac{1}{2}}\right)$. In the extreme (point S) $MRS^B = \left(\frac{\alpha+w_1}{w_2}\right)^{\frac{1}{2}} \left(= \left(\frac{\widetilde{w}_1}{\widetilde{w}_2-\beta}\right)^{\frac{1}{2}}\right)$. On the other hand, when B is on the boundary prices should be equal to $MRS^A = \left(\frac{x_1^A}{\beta+x_2^A}\right)^{\frac{1}{2}} \left(= \left(\frac{x_1^A}{c_2^A}\right)^{\frac{1}{2}}\right)$. In the extreme (point S') $MRS^A = \left(\frac{w_1}{\beta+w_2}\right)^{\frac{1}{2}} \left(= \left(\frac{\widetilde{w}_1-\alpha}{\widetilde{w}_2}\right)^{\frac{1}{2}}\right)$. Therefore the prices will lie between $\left(\frac{w_1}{\beta+w_2}\right)^{\frac{1}{2}}$ and $\left(\frac{\alpha+w_1}{w_2}\right)^{\frac{1}{2}}$. Note that $\left(\frac{\beta+\omega_2}{\alpha+\omega_1}\right)^{\frac{1}{2}}$ is in between. In parenthesis are the answers using the "new" variables.

(c)

WE price ratio is unique. One way of seeing that is to notice that for any feasible endowment the budget lines can not cross in the interior of the box. For endowments between the dotted lines with slope $\frac{\beta+\omega_2}{\alpha+\omega_1}$ this is clear, because all the budget constraint are parallel. For endowments in the triangles ASB and $A\prime S\prime B\prime$ this is also the case because as we move from A (A') to S (S') the slope of the budget constraint (i.e. the prices) is going down (up).

3 Q3

(a)

The problem is

$$\max \left\{ pq - wL^d \mid q = \left(L^d\right)^{\frac{1}{2}} \right\} \tag{2}$$

The FOC is

$$\frac{1}{2} p \left(L^d\right)^{-\frac{1}{2}} = w \tag{3}$$

Then the labor demand is

$$L^d = \left(\frac{1}{2} \frac{p}{w}\right)^2 \tag{4}$$

(b)

The problem is

$$\max \left\{ C^2 \left(24 - L^s\right) \mid pC = wL^s \right\} \tag{5}$$

or

$$\max \left\{ \left(\frac{w}{p} L^s\right)^2 \left(24 - L^s\right) \right\} \tag{6}$$

Taking logs, the FOC is

$$\frac{2}{L^s} - \frac{1}{24 - L^s} = 0 \tag{7}$$

Then the labor demand is

$$L^s = 16 \tag{8}$$

(c)

In equilibrium

$$L^s = 16 = \left(\frac{1}{2} \frac{p}{w}\right)^2 = L^d \tag{9}$$

Then $\frac{p}{w} = 8$.

(d)

By Walra's we know that if there are n markets, we only have to worry about $n - 1$, because in a WE if there is n markets and $n - 1$ of them are in equilibrium the last one must also be in equilibrium.

In this economy there are two markets: labor and good markets. So if labor market is in equilibrium the good market should also be in equilibrium.

You can see it directly. At $\frac{w}{p} = \frac{1}{8}$, labor is 16 and then the production is 4. Profit (an hence owners consumption) is equal to production less total cost, that is $4 - 2 = 2$, and workers consumption is $\frac{w}{p} L = 2$. So demand equal to supply.